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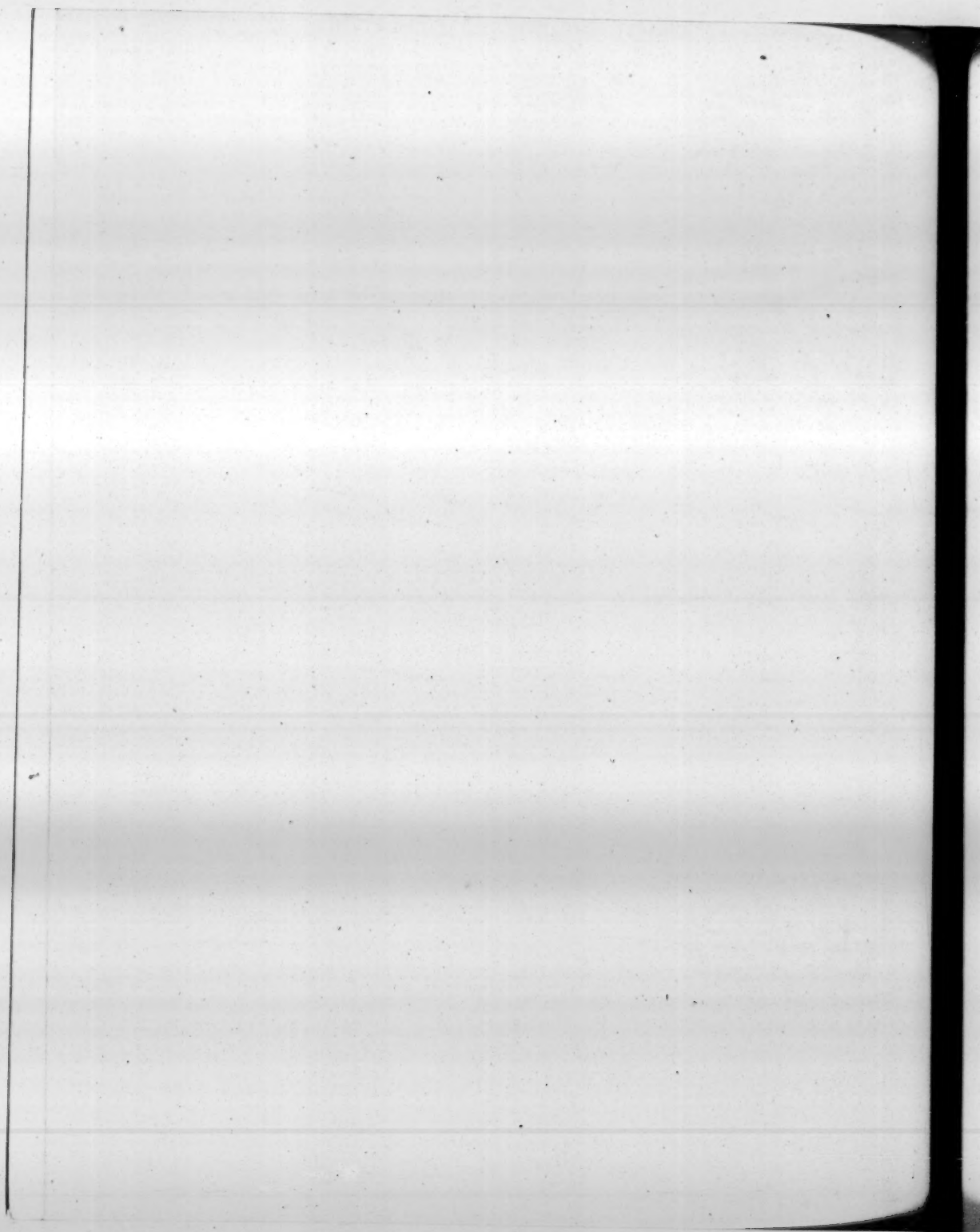
OF THE

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XXXIV.—*A Biographical Notice of the late THOMAS CHALMERS, D.D. & LL.D.*

By the Very Reverend E. B. RAMSAY, M.A., F.R.S.E.

(Read 4th March 1849.)

MR PRESIDENT,—It has been a practice from the foundation of the Royal Society of Edinburgh, to commemorate its deceased distinguished members by memoirs or biographical notices, read at the ordinary meetings of the Society. Some of these have been printed in the Transactions; and our published volumes are enriched by papers of DUGALD STEWART, Professor PLAYFAIR, Sir JOHN MACNEIL, and Dr TRAILL, on the characters and writings of ADAM SMITH, Dr HUTTON, Professor ROBISON, Sir CHARLES BELL, and Dr HOPE. A biographical notice is now due to the memory of a distinguished countryman, late Vice-President of the Royal Society; and the following remarks will, in attempting that object, make a deviation from those more severe discussions with which the time of the Society is usually occupied, in connection either with pure mathematics, natural philosophy, or natural history.

I consider it scarcely becoming for the reader of a paper to occupy the time of the Society, by details or explanations which are merely personal. I would, however, ask permission to state, that I did not enter upon this office till I knew that it had been declined by one far better qualified for its performance; one who, if named, would, I am confident, be recognised as the individual of our body best calculated to do justice to the subject.

I feel assured, however, that, from those whom I have the honour to address, I shall receive every sympathy and indulgence in the few observations which I propose to offer in attempting to delineate those literary characteristics—those efforts of practical benevolence—by which the subject of this brief notice was distinguished during the many years which, as a public man, he came before his contemporaries.

THOMAS CHALMERS was born at Anstruther, 17th March 1780, and at its parochial school received his early education. He studied at the University of St Andrews the usual course of eight years, from 1791 to 1799. He received licence from the Presbytery of St Andrews, 31st July 1799. During the sessions 1799-1800, 1800-1801, he studied at Edinburgh under Professors ROBISON, STEWART, and HOPE. He commenced his clerical life as assistant at Cavers, December 1801—was instituted to the Parish of Kilmany, Fife, 12th May 1803—removed to Glasgow, 1815—to St Andrews, as Professor of Moral Philosophy, 1823. He came to Edinburgh as Professor of Divinity, 1828, and filled that chair till the Disruption in 1843. In February 1834 he was elected a Fellow of the Royal Society, Edinburgh—in 1835 a Vice-President. In January 1834, he was elected a corresponding member of the Institute of France, before which distinguished body he read, in 1838, a paper, in

English, on the distinction between legal charity for the relief of indigence, and legal charity for the relief of disease. At the annual commemoration of Oxford, 1st July 1835, he received the honorary degree of LL.D. In that ancient and Episcopal seat of learning this degree was conferred upon the Presbyterian Professor amidst enthusiastic acclamations, without one dissenting voice. His death took place, 31st May 1847, at the age of 67: He was buried, 4th June 1847.

Of a life so long extended, and embracing so many subjects of active exertion, it is evident such a paper as the present can include only a very abridged and limited notice. It is not intended to embrace those points which belong to mere personal and private biography, or to details of questions on which there existed special and peculiar relation to his own religious communion. There is, I believe, in preparation a full Life of Dr CHALMERS, which will include a publication of his private memoirs, of his correspondence, and other personal biographical expositions. We have now to consider Dr CHALMERS as he came before the world, as he occupied a distinguished place in the observation of mankind; for his reputation was not merely Scottish, or merely British,—it was European. In this view, then, I think we may at once, for the sake of preserving something like method and order in our remarks, consider his public character under three heads:

1. As an Author.
2. As a Political Economist.
3. As a Speaker.

First, One thing strikes us at first approaching the subject of Dr CHALMERS' writings, and that is, the great *industry* which must have marked his literary labours. When we look at the array of volumes published during his lifetime; when we consider the manuscripts which he left behind; and, in addition to all this, take into account that these volumes were not written in the retired cloisters of a college, or the quiet of a country parsonage, but that he wrote in the bustle of numerous engagements, of meetings to be attended, of lectures and examinations for his classes, of correspondence to be maintained, and perhaps, above all, amidst lavish encroachments made upon his time by strangers; we must be struck with his economy of time, and with the *perseverance* of his mental efforts. How many might say of him, as the Younger PLINY wrote of his uncle, the Elder PLINY, "*Erat incredibile studium summa vigilantia. Itaque soleo ridere, cum me quidam studiosum vocant; qui si comparer illi sum desidiosissimus.*"* Dr CHALMERS was far from being, in the classical or scholastic sense of the term, a *learned* man, or a great scholar. His early education, his habits, and pursuits through life, prevented it.† But it is a pleasing

* Plin. Epist. iii. 5.

† In his Lectures on the Romans, he makes no reference to an exegetical or critical view of the passages, though in that Epistle there is a great temptation to do so. He takes the statements of the Apostle in their broadest and most general acceptance. His mind did not rest on the niceties of philological distinctions.

view of his character to find how much he admired and respected learning in others. He never undervalued an attainment because he did not possess it himself. He impressed his students with the value and importance of learning in Theology, and revered what he called the "massive erudition" of divines of the English Church. In describing the peculiarities of his mental constitution, we are at once led to the conclusion of a remarkable predominance of one, and that is an extraordinary abundance of the *imaginative faculty*,—the power of illustrating his ideas, and of setting forth his subjects of discussion with never-ending variety of imagery, comparison, and analogy. In some of his works it seems as if he could not tear himself away from the pleasure of reproducing some great truth, which he enforces under all the different garbs and attitudes with which he can invest it. There is no question that this is a very effective and important method of handling subjects, when the particular bent of the author's genius enables him to pursue it effectually, and is specially adapted for leaving a clear, distinct, and vivid impression upon the mind. In the case of Dr CHALMERS, attachment to science, and early pursuits in astronomy, chemistry, and other branches of physical science, gave him a great advantage in furnishing types for analogy and illustration. These he used on some occasions with happy effect. Indeed, he never lost his interest in the exact sciences; and, had the circumstances of life been favourable to their pursuit, would, no doubt, have been distinguished in the branches of mathematical pursuits. His mind was always alive to scientific subjects. In 1838, when introduced to the present Bishop of Nova Scotia, he heard, with much interest, the Bishop's description of the Bay of Fundy (which is in his diocese), and the enormous roll of tide coming in with a front 70 feet in height; next day Dr CHALMERS wrote a letter to the Bishop, proposing the experiment of having a delicate pendulum placed on the shore, and to watch the effect of the mass of water upon it, as they came into the bay, similar to Dr MASKELYNE's celebrated experiment at Schehalion, to test the effect of gravity, but, with the advantage over Dr MASKELYNE, that the waters would form a homogeneous mass of matter, and the result be more striking, from marking the effect of the mass *approaching* the pendulum.* When I said, therefore, that, in Dr CHALMERS, the faculty of imagination was an abounding and prominent endowment, I was far from meaning that this implied a poverty of the reasoning faculties, or defect in other mental qualities. On the contrary, he had a mind remarkably adapted for the apprehension of great principles, of broad and profound truths. He delighted to grasp primary and fundamental elements. He expatiated, with the fullest enjoyment, on reasonings of such authors as Bishop BUTLER, BACON, NEWTON. His admiration of BUTLER was intense: as an expounder of great elementary truths, he placed him in the first and highest class

* This experiment, I find, had been suggested by Professor ROBISON, in his *Elements of Mechanical Philosophy*, § 474.

of human intellects. In the dedication of his *Bridgewater Treatise* to the BISHOP of LONDON, he thus expressed his admiration: "I have derived greater aid from the views and reasonings of Bishop BUTLER, than I have been able to find besides in the whole range of our existent authorship." On one occasion, when some person present was animadverting upon the wealth of the Church of England, and gave, as an example of its over-abundance, the revenues of the See of Durham, the Doctor exclaimed, with characteristic eagerness, "Sir, if all that has been received for the Bishopric of Durham since the foundation of the See, were set down as payment for BUTLER'S *Analogy*, I should esteem it a cheap purchase." We are not to consider his admiration of BUTLER'S works as proceeding from the sameness or resemblance of their mode of reasoning, but rather from the difference. BUTLER excogitated masses of profound thought, and left them nearly as raw material, costly indeed, but not elaborated for use, except for the purpose of furnishing him with examples of *analogy* between natural and revealed theology. CHALMERS found, in this storehouse, abundant substance for practical application to the business and improvement of life. He polished and carved, and adjusted the stone which he had dug from the quarry. And thus, both as an able quarryman, and as an accomplished dresser, he has erected graceful, durable, and useful edifices for mankind. His method of exhibiting truths, in so many and in such attractive positions, has deeply impressed the minds of thousands, not only of those who were amongst his stated hearers as pupils, but amongst readers of his works generally. Although Dr CHALMERS' mode of treating his subjects was such as I have described, and though his usual mode of handling was to exhibit *one* great and leading topic, illustrated and enforced with all the profusion and imagery of a rich fancy and a powerful imagination, we should, at the same time, observe that the method is frequently applied with great ability, and with great effect in bringing forward *two ideas* where one is required to check or modify an exclusive attention to the other. Thus, for instance, in his Sermons, though he dwells upon the doctrine of the corruption of human nature, and the utter insufficiency of all mere natural efforts to merit the Divine favour, and to claim a reward at God's hand, he runs, as it were, parallel with this great truth another truth, equally important and equally authoritative, viz., that virtue in itself is beautiful, that the generous affections and good feelings must not be undervalued or depreciated, but are, in fact, desirable and estimable in their own place and their own character, and require only the right *motives* to render them acceptable. I know no writer who has more successfully elaborated this important subject. He has shewn the harmony and consistency of the two doctrines. He has upheld and vindicated the dignity and the loveliness of virtue. He has cut away all ground of merit and of human desert before God. In the same manner, as a predestinarian, he has ably and powerfully (in some instances sternly) put forward the proofs of God's pre-science and omnipotence over all his works; but, in conjunction with that great truth, he has upheld, with unflinching fidelity, the necessity of human exertion,

and he has illustrated the agency of man's own endeavours as fully and as freely as if he had been the champion of a free will entire and uncontrolled. Thus it is always in his writings. He is urging and reiterating, with all the fervency of an ardent eloquence, a great and important principle, or he is running the parallel between two essential truths. He is sustaining, singly and conjunctly, the position of two considerations, both of which are to be of supreme authority. The action of both is requisite for man's moral and spiritual wellbeing; at times they may, in theory, appear to be incompatible, but in action are never inconsistent. He is not, therefore, a writer of subdivisions or details. He is copious, but copious in illustrating great propositions. He offers, in this respect, a remarkable contrast to a great writer, Dr ISAAC BARROW, whose strength is in *division*. Of him it was said, that he "*exhausted* his subject." CHALMERS also exhausted his subject. But then one exhausted the practical application and minute enforcement of a truth, in all its results and consequences; the other exhausted the various forms and illustrations by which that truth itself could be enforced upon the human mind. There is nothing of the analytical method in his treatment of a subject. It is almost purely *deductive*. He sets out with a great principle, and shews, in a thousand shapes, its application and appropriation. One remark, however, we would make on this subject. Although the handling is so copious and diffusive, it is seldom deficient in strength and pungency. It would frequently be difficult to abbreviate without injury; and we find expressions constantly occurring of great force and point. It was said of Dr CHALMERS by ROBERT HALL, after hearing him preach, that his sermon went on hinges, not on wheels. Images are sometimes dangerous coadjutors. A discourse on wheels may run off the course; but a discourse on hinges must, at any rate, retain the speaker in his place, and make him exhibit the various forms and phases of his subject, by turning it in every direction to his audience.

The style of Dr CHALMERS' writing partakes of the character of his mind. It is copious and overflowing; cumbrous, perhaps, at times, for the more minute detail of a subject; but the phraseology (though occasionally somewhat eccentric) is often powerful and beautiful in the highest degree. It is impossible to illustrate these peculiarities without examples. I shall only select a few. Thus, to express the quick passage of time: "Time, with its mighty strides, will soon reach a future generation, and leave the present in death and in forgetfulness behind it." To express that the world occupies our thoughts; "Its cares and its interests are plying us every hour with their urgency." A man of shallow views in religion is a "man whose threadbare orthodoxy is made up of meagre and unfruitful positions." The external marks of piety: "A beauty of holiness, which effloresces on the countenance, and the manner, and the outward path." To say that the repentance of a sinner interests the angels, is thus worded: "His repentance would, at this moment, send forth a wave of delighted sensibility throughout the mighty throng of their innumerable legions." Persons who take

their opinions from a *partial* adoption of Scripture truth, are persons who, "retiring within the entrenchment of a few verses of the Bible, will defy all the truth and all the thunder of its warning denunciations."

His style, with all its peculiarities, was HIS OWN. It may be called mannerism; but it is the mannerism of a powerful mind striving to express its own conceptions without regard to rules of rhetoric or the discipline of schools. It is the mannerism of genius,—one leading characteristic of which is to invest known truths and ordinary objects with new and untiring interest, and with constantly fresh attraction; and, on this ground, it is characteristic and becoming, because it is his own; and, accordingly, these peculiarities of style pervaded his ordinary conversation and his familiar letters, just as much as they marked his more elaborate compositions; and in the ordinary intercourse of life, expressions constantly recurred to remind one of his writings. In fact, his language is merely the vehicle or medium of expressing and communicating his ideas; and we may almost say *he could not help it*. There is a danger with him (as there is with all imaginative writers) of his style being considered imaginative *only*. To many minds declamation is irksome and wearisome in the highest degree,—to them it conceals rather than develops the mental power which lies below the surface; and, not unfrequently, practical wisdom and sound argument are not duly estimated, simply *because* there is a play of imagination around them,—the lustre and richness of the setting obscures the pearls. Such authors are not unfrequently a snare to their admirers. Mannerism in authors may be easily caught by those who have no inspiration of their genius. Hence, of all writers and speakers Dr CHALMERS was one most dangerous for imitators (and amongst young and injudicious students he had imitators). What was natural to him was constraint or affectation in them. In fact, they became copyists more than imitators. Their taking his style and manner becomes a literary larceny, rather than an honourable and fair obligation. It is miserable to see men borrowing fine clothes which they know not how to wear,—affecting a glow of eloquence to cover a vapid and commonplace conception of their subject.

Secondly, As affecting the happiness of mankind, and as bearing upon their best and highest interests for time and for eternity, Dr CHALMERS was, during the whole of his public career, much occupied with the theories of Political Economy. In all ages of the world, how much of the misery of mankind may be traced to the errors and mistakes of erroneous legislation. Bad laws on excise,—on poor management,—on taxation,—on police or criminal jurisprudence, proceeding from false views of political economy, have been the most fruitful sources of crime, of misery, and degradation. The energetic and benevolent spirit of CHALMERS saw and felt the connection between a well-doing and a well-living population. He felt how much, under the Divine blessing, might be done by rulers and statesmen

to make or mar human happiness, and he took a very prominent position amongst the Christian economists of the day. Into the general question of political economy as a theory, whether of population, free trade, balance of trade, capital, taxes or tithes, I do not pretend to enter. On these points Dr CHALMERS wrote with much power and acuteness. His views on most points generally coincided with ADAM SMITH, MALTHUS, TOOKE, and authors of that school. But in one department of political economy, he took that position which has added lustre to his name, and which exhibits him to the world as the true Christian philanthropist, and the best friend of human nature. Speculations on theory and doctrine in political economy were not sufficient for one who constantly sought to do good to those who most needed the help and guidance of their fellow-Christians. We have to consider CHALMERS, then, as a practical economist; as one who, not satisfied to reason and to speculate in his study upon the best methods of improving the conditions of mankind, went forth into the cottages, the hovels, and crowded habitations of the poor, to improve their temporal, moral, and religious condition. The agencies on which he depended for improving mankind were the school, the Bible, the visitor, the pastor. Hence the titles of his works and articles on this subject, indicate what were the objects and purposes he had in view: for instance, we have "The Civic and Christian Economy of Great Towns;" "The Christian and Economic Polity of a Nation;" "Sabbath Schools;" "Bearing of Christian Economy upon Pauperism," &c. In his "Civic and Christian Economy of Large Towns," he lays down some of the most valuable and practical principles of useful charity. It is a dreary and heart-sickening prospect which the Christian philanthropist encounters when he enters upon the charity of great cities; and not only did Dr CHALMERS zealously promote amendment in that field of our erring, and destitute, and suffering countrymen, by suggesting sound principles of management, but he threw his whole energy, his persuasive eloquence, and his personal superintendence into the work.*

In 1815 he had been called to take the pastoral charge of a parish in Glasgow, a city where he knew there would be abundant opportunities for verifying his opinions and employing his resources. He commenced the publication of *The Civic and Christian Economy*, as a small periodical, and took the lead in directing the attention of the nation to the absolute necessity of extending, in our city population, means of education, of pastoral superintendence, and spiritual instruction, *similar to what prevailed through the country parishes*

* It is pleasing to remember how the last mortal days of such a man were engaged with plans of instruction for the benefit of this very class. He had for some time been entirely taken up with a School and Church, in the worst locality of the Old Town of Edinburgh. The man of high speculation became a teacher of ragged children. The Professor of Theology descended from his chair to impress the first rudiments of Christian truth upon the rude minds of a congregation the most ignorant and most neglected.

of Scotland. However ample and effective had been the supply of these elements of human improvement in the agricultural parishes and districts, the commercial and manufacturing population had quite outgrown them, and the work required to be recommenced, and taken in hand in good earnest. He was, therefore, a strenuous and constant advocate for carrying out the system of TERRITORIAL SUBDIVISION. There was a vitally important principle in the accomplishing this great end, and one which Dr CHALMERS established with great ability: it was the principle of providing for the work being *effectually* done, in the particular portions or districts chosen—not only the taking in hand the worst localities, but in every one of these laying a sufficient foundation or substratum of good, so far as you go. I think this principle was first taken up by Dr CHALMERS. It is of immense importance, and I know was adopted from Dr CHALMERS by the BISHOP of LONDON, in consequence of consultation with him regarding the plans for providing churches, schools, and parsonages, for the recently-formed masses of the destitute population of the great metropolis. The experiment was tried in Bethnal Green, where ten new parishes were formed, dividing the population into sections manageable by a pastor, and curate, and school. For want of attending to this principle, a grant of a million of money for church-building in England had been rendered comparatively ineffective. Churches and schools were set down here and there; lost in the mass of surrounding poverty and destitution, their influence was little felt,—in some cases almost unnoticed.*

I have now to notice, in connection with the political economy of Dr CHALMERS, an important incident of his life. And I must allude to an achievement which exercised the greatest influence upon his own views of the parochial system and management of the poor, and which excited astonishment, admiration, and scepticism amongst his contemporaries. I refer to the remarkable effects produced by management of the poor in St John's Parish, Glasgow, under his direction and superintendence. I will endeavour to make a plain and distinct statement of the FACTS, as established by the evidence of the parties concerned in the operation.

It is well known how exceedingly Dr CHALMERS was opposed to the support of the poor by a *compulsory assessment*; that is to say, the *ordinary* wants and the *ordinary* support of the poor. He approved of assessments for disease and casualties, for supporting infirmaries, dispensaries, and lunatic asylums, also for extraordinary emergencies of famine, pestilence, or catastrophe; but general poor-

* This principle of territorial subdivision, for which Dr CHALMERS, as a Christian philanthropist, so long contended, is at last acknowledged as the essential preparation for bringing spiritual instruction to bear upon the worst portions of our crowded and demoralised population. Lord ASHLEY, the enlightened friend of the poor, has, with the full approbation of the Premier, moved for a commission to inquire into the best method of dividing all parishes in England which contain a population of 10,000 or upwards.

laws he utterly condemned. He had termed the system a "legalised enormity." He had ascribed to the action of those laws in England all the evils under which the country suffered from pauperism. He considered them to be the bane of Christian charity, and the curse of all connected with them. It remained, then, to test by experience, when he had a proper field, an *opposite* system; and this he was determined to do in Glasgow. When, in the year 1815, he took charge of the Tron Church Parish in Glasgow, the system of management for the poor throughout the city was somewhat peculiar. The whole funds raised for the poor, whether in the shape of assessments or collections at the church-doors, were under the administration of two bodies, one called the General Session, consisting of the elders and clergy of all the parishes, and the other called the Town-Hospital, which had pensioners within its walls, and *out*-pensioners residing in the city. The whole expense of poor support had been on the increase. In 1803 it amounted to about £4000; in 1818, to about £11,000; in 1820, to £13,000. His determination was, from the first, to manage his district *without* assessment. In this wild and extravagant scheme, as it was considered, he was opposed by the General Session, by the Magistrates, by the Town-Hospital, and by the Presbytery. Indeed, the Presbytery had carried up a case against him to the General Assembly; accordingly, he was glad to be transferred to St John's Parish, which took place in 1819, and where the same obstacles and impediments to his experiment did not exist. The population was 10,000; the people, with very few exceptions, of the poorest class of manufacturers. According to the due proportion of population and pauperism, the expenditure for St John's had been about one-tenth of the expenditure for all Glasgow, or upwards of £1400 annually. His first step was to release the General Session and the Town-Hospital from all obligation to support the St John's poor, and he undertook, with his own church-door collection, to meet their wants. This collection averaged £400 a-year. With £400 a-year, therefore, he began the work. Now, of this sum, £225 were already pledged for regular cases permanently settled upon parochial relief, so that, from this collection fund of £400, £175 only remained as a *surplus* to meet and to provide for *new* cases of pauperism. But, besides the £400—the result of day collections at the church-door—there was another and an evening collection made by a very poor congregation, chiefly in halfpence, which amounted to about £80 a-year. Out of this £80 he resolved to provide for new cases of paupers coming upon the parish, and to leave the £400 collection to take care of the old paupers. He had previously made a minute district subdivision of the parish, and secured the assistance of zealous and intelligent deacons as visitors, one for each district. What, then, was the *result* of the system, and the degree of success with which it was accompanied? The £80 covered the whole expense of the *new* pauperism, which did not require more than £66, 6s. The £400 were, in the mean time, increasing in the hands of the kirk-session by old paupers dropping off, and by the surplus

of £175 not being required. This command of money in the hands of the kirk-session Dr CHALMERS considered to be a snare and a danger; accordingly, as he expressed it with considerable *naïveté*, he sought to provide "a safe and salutary absorbent" to take off this plethora of pecuniary oppression, and this he did by expending it all in the permanent endowment of a school. Thus the system worked, and the only disturbing force seems to have been the occasional indiscreet and injudicious introduction of charitable contributions *from without*: And certainly here is a marvellous result,—the poor of a parish absolutely managed with a success varying *inversely* as the pecuniary resources at the command of the managers. But neither the principal mover of this scheme, nor his colleagues in the work, seemed to consider it a mystery or a miracle; their solution of the problem was; 1st, that former applicants who were conscious that they did not require or deserve support withdrew, and the idea of legal right ceasing, no cases but those of absolute necessity were left; but, 2^d, and chiefly, that the sympathies of the poor themselves were thus called forth, and no one allowed his neighbour to starve so long as he could spare a morsel, and when he knew that neighbour was deprived of other resources on which he could depend. The poor, in short, helped each other through their difficulties when no one else would. The artificial channels of charity being closed, a more copious and more permanent supply flowed through the natural channels of relationship and vicinage. Such was the theory; the results were indisputable. The world was still sceptical, and two solutions were offered to account for the success of a scheme which would support poor people without poor-laws. It was said, in the first place, that the system was so hard upon the people that the poor were driven out of St John's parish, and took refuge in other parishes, where more money was expended. It was said, in the second place, that the success was the consequence of Dr CHALMERS' personal influence and powers. That what he accomplished in St John's, another man *could not* accomplish in St Luke's; and that, with the man, the scheme would die out. To both of these objections an answer was ready. To the first objection it was declared, that the balance of migratory pauper population was fully in favour of St John's; and, to come to greater exactness, it was stated that a correct account was kept of poor *leaving* St John's, and poor *coming in* to St John's: the result was the imports *exceeded* the exports by fourteen souls. The exchange, in fact, was *against* them, and this they considered a conclusive answer to the charge of harsh treatment of paupers. To the second objection it was replied, that the system worked for many years *after* Dr CHALMERS' departure from Glasgow, and succeeded also in other manufacturing parishes of Scotland where it was tried—the Gorbals of Glasgow and Langholm being cited as favourable examples. How it was that, in the face of an experiment apparently so successful, detailed by himself in evidence before a parliamentary Committee, a more stringent enactment of poor-laws for Scotland should have been made, and the system be adopted for

Ireland ; or how, in the public mind, it did not produce a stronger feeling *against* compulsory charity in general, I am not competent to decide. The facts are indisputable, and were, during the whole of Dr CHALMERS' lifetime, after he left Glasgow, referred to in corroboration of the correctness of his theory, and as a standing proof that charity, if left to itself, *would* supply means for the maintenance of the poor, and a maintenance of a more suitable and effective nature than could be done by a compulsory assessment. In all his treatises on Management of the Poor, he alludes with unshaken confidence to the great Glasgow experiment.

The complete and detailed account of the experiment will be found in four articles, forming the general Appendix on Pauperism, in the sixteenth volume of his collected Works, including his own evidence before the Committee of the House of Commons on the subject of a poor-law for Ireland. Great prejudice existed (in England especially) against the whole system, as harsh, and severe, and cruel, and numerous objections were urged against the possibility of success. One objection brought by the writers of articles on Poor-laws in the Quarterly Review, against the plan of withholding an assessment for supporting the poor, and throwing them on the natural or voluntary principle of charity, was an unjust one, and indicated a misapprehension of the whole system upon which that method was grounded. It was said that the principles advocated by Dr CHALMERS were an encouragement to vagrancy and mendicity. Therefore, according to this view, it was merely a question whether we were to have parish *paupers* or highway and street *beggars*. But the writers of those articles did not consider that on no point was Dr CHALMERS' views of pauperism more decided than on the *discouragement* of relief to common vagrants and beggars. The principles on which the Glasgow experiment was accomplished, when carried through, would have entirely put down common beggars ; and Dr CHALMERS drew an ingenious and novel argument *against* promiscuous charity from the example of our Lord, as recorded in the four Gospels. He healed all diseases and sickness in those who came to him ; but only on two occasions did he supply by miracle the multitudes with *food*. These were occasions of urgency ; and when he found that they came to him idly and *on account* of food, he firmly withheld it.

But, Sir, I would now turn to another subject connected with the great question of a nation's civic economy—and that is the Endowment of its Church and Universities. On these points Dr CHALMERS has written with remarkable force and much enthusiasm. And he has propounded the compulsory endowment theory for ecclesiastical and educational objects as vigorously as he has disclaimed it for sustaining the poor. His essay "On Ecclesiastical and Academical Endowments" has been described in the Quarterly Review (vol. xlv., p. 527) "as one of the most vigorous and eloquent defences of such endowments that ever proceeded from the press—a treatise which would alone have been sufficient to immortalize its author." This is high praise from such a quarter : But I think it is

deserved, and fully deserved. There is great power of argument, felicitous illustration, and a glowing enthusiasm of admiration, for the theological literature, and the erudition, and the learning, and the eminent men produced by the ecclesiastical and academical endowments of England. In reference to the Church of England he writes:—"There are many who look with an evil eye to the endowments of the English Church, and to the indolence of her dignitaries. But to that Church the theological literature of our nation stands indebted for her best acquisitions; and we hold it a refreshing spectacle, at any time that meagre Socinianism pours forth a new supply of flippancies and errors, when we behold, as we have often done, an armed champion come forth in full equipment, from some high and lettered retreat of that noble hierarchy; nor can we grudge her the wealth of her endowments, when we think how well, under her venerable auspices, the battles of orthodoxy have been fought,—that in this holy warfare they are *her* sons and *her* scholars who are ever foremost in the field—ready at all times to face the threatening mischief, and by the weight of their erudition to overturn it."

In the same work, "On the Use and Abuse of Literary and Ecclesiastical Endowments," he thus writes of Oxford and Cambridge:

"We cannot conclude this passing notice of the Universities of England, without the mention of how much they are ennobled by those great master-spirits, those men of might and of high achievement,—the Newtons, and the Miltons, and the Drydens, and the Barrows, and the Addisons, and the Butlers, and the Clarkes, and the Stillingfleets, and the Ushers, and the Foxes, and the Pitts, and Johnsons, who, within their attic retreats, received that first awakening, which afterwards expanded into the aspirations and the triumphs of loftiest genius. This is the true heraldry of colleges. Their family honour is built on the prowess of sons, not on the greatness of ancestors; and we will venture to say, that there are no seminaries in Europe on which there sits a greater weight of accumulated glory, than that which has been reflected, both on Oxford and Cambridge, by that long and bright train of descendants who have sprung from them. It is impossible to make even the bare perusal of their names without the feeling, that there has been summoned before the eye of the mind the panorama of all that has upheld the lustre, whether of England's philosophy, or of England's patriotism, for centuries together. We have often thought what a meagre and stinted literature we should have had without them; and what, but for the two Universities, would have been the present state of science or theology in England! These rich seminaries have been the direct and the powerful organs for the elaboration of both; and both would rapidly decline, as if languishing under the want of their needful aliment, were the endowments of colleges swept away. It were a truly Gothic spoliation; and the rule of that political economy which could seize upon their revenues, would be, in effect, as hostile to the cause of sound and elevated learn-

ing in Britain, as would be the rule of that popular violence which could make havoc of their architecture, and savagely exult over the ruin of their libraries and halls."

Now, throughout the whole of this Essay on Endowments, and in the lectures which he delivered with so much success in London before Princes of the Blood Royal, Peers, Bishops, Ministers of State—the highest and the most intelligent of the land—it will be observed that he constantly advocated compulsory enactment or permanent endowment for support of the objects on which he lectured. He maintains this opinion chiefly on the ground, that individuals are not in all cases the best judges of their own interests, and will not always voluntarily employ their means in that way which is most conducive to their own benefit and that of society. In religion the supply must not be delayed till the demand come forth to claim it. The demand is, in fact, to be created, for there is no natural appetency for religious instruction; and so, as he himself declares, "the great argument for *literary* endowments is founded on the want or weakness of the natural appetency for *literature*." Now the difficulty which most people have in following Dr CHALMERS' views on pauperism, arises out of this very argument of his own in defence of academical and ecclesiastical endowments. For may it not be urged, if the principle of provision by compulsory payment be so clear and applicable to the case of sustaining ecclesiastical and academical institutions, why is it not equally applicable to provision for maintaining the poor? The natural appetency for *charity* is frequently quite as dull and torpid as natural appetency for religious or literary instruction. As a high and moral obligation, should it not therefore also be compulsory equally with the others? But the poor do assist each other in their poverty. But then, again, it may be asked, why should the support of the poor be *confined* to the poor? They see their brethren suffer, and charity is forced upon them. The more wealthy neighbours live at a distance. If human distress were forced upon *their* notice, *they* too would help. But they do not witness suffering at their doors, and so they forget it. But ought they to be allowed to forget it? Whatever force there may be in these or similar arguments, one thing is clear, the Glasgow experiment did not practically convince the Legislature that they might now abandon all compulsory assessment for the poor, and throw themselves upon the natural charity of mankind for better attaining, *without* compulsion, the same object. This, however, be it remembered, is no real argument either against the truth of the statement or the soundness of the theory. The highest exercise of Christian charity is undoubtedly the voluntary; indeed, giving to the poor except voluntarily, is not charity at all. The principle may be pure and right, but human nature is not perhaps yet fitted to receive it, or capable of acting upon it. A time may come when the world will discern and receive it, when the outpourings of Christian love to the brethren will so promptly and so amply supply all the wants of the poor, that assessments will

be unheard of. Men will do that on principle which now they must do by legal enactment. Such a state of things would follow the universal prevalence of Christian charity in men's hearts, and is not therefore to be considered a mere chimera. Should this triumph of principle and of love ever be achieved amongst mankind, what will be said and thought in *those* days of the mind that, amidst scepticism and ridicule, had resolutely maintained the principle, nay, which had in its own sphere of action practically worked out its successful application?

Thirdly, And now, Sir, we have to consider Dr CHALMERS as an orator. He was distinguished as a preacher, as a speaker at public meetings, and as a member of ecclesiastical courts. We attribute to him in all these positions, especially in the pulpit, the quality of a high and a peculiar *eloquence*, and we have the utmost confidence in the correctness of this estimate; for if CHALMERS were not eloquent, where, we may ask, is eloquence to be found? Judge by the effects upon men's minds, and say, is not that eloquence which captivates and enchains the hearers? Is not that eloquence which delights all classes of mankind, all ages, all situations of life? Is not that eloquence which ensures an interest and admiration unbroken, and which to the last attend every appearance of the speaker in public? Nor was this attraction the result of art, or the merely artificial embellishments of oratory. It was not in graceful and studied action. It was not in musical and practised intonation. It was not in the purity and beauty of the accent. All these were plain, homely, to some hearers quite unusual; and yet how extraordinary were the effects of his eloquence! Such effects, then, being the result, not of artificial embellishments or natural grace of manner, tones of voice or skilful action, are attributable to the power and energy of the preacher's own spirit, to the vivid pictures which he brought before his hearers, the fervid oratory with which he took captive the heart and understanding. One important element of his success as a preacher, I think, was the impression of earnest truth and sincere conviction existing in his own mind. As to the mode of arguing and the style of composition, the remarks already made upon Dr CHALMERS as an author, apply to him as a preacher. Indeed, all his writings seem as if composed for *spoken* communication, and the method is favourable to producing one vivid and powerful effect upon the mind. No one indeed, who has not *heard* Dr CHALMERS in his day of vigour, can form a correct idea of his power as a pulpit orator. It is now thirty years since his Astronomical Sermons were delivered, and though I suppose no discourses ever produced a greater effect, the nature of that effect must be little known to the younger members of the present generation. The fame of a preacher mainly depends (like the fame of an actor or singer) upon traditionary description. In many cases, the perusal of written discourses gives little notion of the effect in delivery; in some cases, as of WHITFIELD, Dean KIRWAN, and other eminent preachers, who, in their day, produced marvellous sensations, they give *no*

notion at all ; the effect must have arisen entirely from the *manner*. And when we consider how much pleasure the printed Sermons of Dr CHALMERS now afford to the intelligent reader, we may easily imagine the delight with which they must have been heard, coming with all their novelty and fervour, fresh from the preacher's lips. To enter into any description or analysis of compositions so well known as these published Sermons, would be here quite out of place. I may perhaps refer to one or two passages as specimens, and favourable illustrations of his own peculiar manner. In his sermon "On Cruelty to Animals" (preached in consequence of an endowment), he has occasion to shew that suffering is often inflicted on the inferior creatures by man, not for the purpose of torment, but that it follows whilst he is occupied with other considerations and excitements ; and as an example, to illustrate the absence of any cruel *purpose* for the mere infliction of pain, he described in glowing colours the excitement and the interest of an English hunting-field, and he terms it "this favourite pastime of joyous old England, on which there sits a somewhat ancestral dignity and glory." And he described the "assembled jockeyship of half a province," the assemblage "of gallant knight-hood and hearty yeomen," and he spake of "the autumnal clearness of the sky," and "the high-breathed coursers," and "the echoing horn"—"the glee and fervency of the chace,"—"the deafening clamour of the hounds," and "the dying agonies of the fox," in such a strain of animation, that Lord ELCHO's huntsman, who was present, declared that he had difficulty in restraining himself from getting up and giving a *vue-holla*.

Of a far different character was the scene he drew in the conclusion of a sermon preached for the benefit of a Society in aid of Orphan Children of Clergymen. He described the sons and daughters of a Scottish pastor obliged, at their father's death, to leave the peacefulness of their father's dwelling, and appealed to his hearers for their assistance in behalf of those who were so friendless and so dependent. "With quietness on all the hills, and with every field glowing in the pride and luxury of vegetation, when summer was throwing its rich garment over this goodly scene of magnificence and glory, they think, in the bitterness of their souls, that this is the last summer which they shall ever witness smiling on that scene which all the ties of habit and affection have endeared to them ; and when this thought, melancholy as it is, is lost and overborne in the far darker melancholy of a father torn from their embrace, and a helpless family left to find their way unprotected and alone, through the lowering futurity of this earthly pilgrimage." I heard that sermon, and the tears of the *father* and the preacher, fell like rain-drops on the manuscript.

In his Sermon on the Death of Dr THOMSON, describing in a *picturesque* point of view, the proximity of tenderness and power, of gentleness and strength, in the same human character, he added this happy illustration : "This is often exemplified in those alpine wilds, where beauty may at times be seen embosomed in

the lap of grandeur, as when, at the base of a lofty precipice, some spot of verdure or peaceful cottage home seems to smile in more intense loveliness, because of the towering strength and magnificence which are behind it."

In a very striking Sermon on the "Paternal Character of God," when drawing "the picture of moral and pleasing qualities of mind and affections, *apart* from the love of God, or from the influence of divine grace upon the soul," he adds this beautiful illustration: "There is beauty in the blush of a rose, and there is beauty of a higher character in the blush that mantles the cheek of modesty, and yet there may be just as little of loyalty to God in the living as in the inanimate object."

Of his speaking at public meetings, I had fewer opportunities of judging than I have had of his pulpit discourses. On some of those occasions, he produced great impression. But, perhaps, the most distinguished of such appearances was on occasion of a public meeting held in Edinburgh, in the year 1829, on the subject of a bill then pending in Parliament, commonly called the Catholic Emancipation Bill. Dr CHALMERS, in opposition to the views of the generality of those with whom he usually acted in public affairs, civil and ecclesiastical, was in favour of that emancipation, and of the admission of Roman Catholics, Peers and Commoners, into the two Houses of Parliament. The effects of that speech have been described as something very remarkable. An excitement and enthusiasm pervaded the large and closely-crowded assemblage, seldom witnessed in modern times. I heard our most distinguished Scottish critic, who was present on the occasion, give it as his deliberate opinion, that never had eloquence produced a greater effect upon a popular assembly, and that he could not believe more had ever been done by the oratory of DEMOSTHENES, CICERO, BURKE, or SHERIDAN. And this was a case simply of eloquence. For the speech delivered was not remarkable either as to argument or literary composition. It was reported in the newspapers at the time, but has not been deemed worthy of being included in his collected Works. I shall refer to one incident only connected with his speaking in the General Assembly,—and the result was the more remarkable as the reply must have been unpremeditated. He had spoken very strongly against the principle of a clergyman holding the two offices of Professor and Pastor. It was alleged against him that such opinions were, at any rate, inconsistent in him, inasmuch as he had himself been an aspirant for the Chair of Mathematics, and justified the union of professional and pastoral duty. His answer to the charge was striking,—“I feel obliged,” he said, “I feel obliged to the Reverend Gentleman for reviving my pamphlet, and for bringing me forward to make my public renunciation of what is there written. I now confess myself to have indeed been guilty of a heinous crime, and I now stand a repentant culprit before the bar of this Venerable Assembly.” After stating that he had then certainly maintained that a devoted and exclusive attention

to the study of mathematics was not dissonant to the proper habit of a clergyman, he thus concluded:—

“Alas! Sir, so I thought in my ignorance and pride. I have now no reserve in saying that the sentiment was wrong, and that, in the utterance of it, I penned what was most outrageously wrong. Strangely blinded that I was! What, Sir, is the object of mathematical science? Magnitude, and the proportions of magnitude. But then, Sir, I had forgotten *two magnitudes*, I thought not of the littleness of Time,—I recklessly thought not of the greatness of Eternity.”

An important class of productions and of labours come under this head, and occupy a place somewhat intermediate between the pulpit and the public meeting. I refer to his *Lectures on Moral Philosophy*,—on *Evidences*,—and on *Theology*. These lectures were all composed and written with great care; but he introduced, parenthetically, further explanations and illustrations extempore. The remarks made, on his manner of discussion in the pulpit, apply also to his manner of discussion in the Chair. The same fulness of illustration, the same energetic and irresistible enforcement of some leading and fundamental truth,—the same fervour, and the same sincerity. These did not fail to secure the attention, and to engage the affections, of his class. Many persons, not intended for the ministry, attended these lectures; and we have reason to believe that his discussions on *Evidences*, on *BUTLER'S ANALOGY*, and on *Natural Theology*, have, in this generation, exercised considerable influence upon the supposed sceptical tendencies of the northern mind. I will only adduce one passage in illustration of his lecture style. In his *Lectures on Natural Theology*, he draws an argument in favour of an unquestionable act of GOD in creation, from the geological appearances of the world. The commencement of the present economy, after the destruction of the previous economy, is a convincing argument against the *eternity* of creation. The whole reasoning is ably and ingeniously conducted, and, at the same time, clothed in language of a high and imaginative eloquence. He thus asks, How could the present world, after former destruction, be produced otherwise than by a new and palpable act of creation? “Is there ought in the rude and boisterous play of a great physical catastrophe that can germinate those exquisite structures, which, in our yet undisturbed economy, have been transmitted in pacific succession to the present day? What is there in the rush and turbulence, and mighty clamour of such great elements of ocean, heaved from its old resting-place, and lifting its billows above the Alps and the Andes of a former continent? What is there in this to charm into being the embryos of an infant family, wherewith to stock and to people a now desolate world? We see, in the sweeping energy and uproar of this elemental war, enough to account for the disappearance of all the *old* generations, but nothing that might cradle any *new* generations into existence, so as to have effloresced on ocean's deserted bed, the life and the loveliness which are now before our eyes. At no juncture, we apprehend, in the history of the world, is the interposition of Deity more

manifest than at this ; nor can we better account for so goodly a creation, emerging again into new forms of animation and beauty from the wreck of the old one, than that the SPIRIT OF GOD moved on the face of the chaos ; and that nature, turned by the last catastrophe into a wilderness, was again re-peopled at the utterance of His word."

We naturally feel an interest about the appearance and address, the personal habits and peculiarities, of those who have been distinguished in their day and generation. Such peculiarities, in the subject of this biographical notice, must have been familiar to many now present. For upwards of twenty years I enjoyed the privilege of friendly intercourse ; and it is a pleasing, though melancholy office of memory to recall those traits which rendered his society so interesting, and so delightful. I think I can safely say I never left his company without having some sentiments or expressions in my mind which I felt were worthy to be remembered. There was a mixture of guileless simplicity and acuteness, of playful humour and vigorous conversation, of urbanity and earnestness, which cannot be forgotten. His face was at times radiant with benevolence and kindly feeling. Like many powerful and striking countenances, the expression was chiefly in the mouth. The eye was dull, and often inanimate,—this, in combination with the massive brow, rendered the play of the lower part of the face the more striking ;—on those occasions especially, when, after being silent and apparently abstracted, he would burst forth into some strain of admiration, or some strong expression of his opinion regarding the topic of conversation, or not unfrequently some humorous or ludicrous combination of thought. His habits were social—he was hospitable, and enjoyed the hospitality of his friends. Though, in his whole demeanour, utterly inartificial, he was eminently courteous and pleasing in his address. Though as plain and unpretending in his manners as possible, no man had a more acute perception of refinement of manners in others. I recollect his *enthusiastic admiration* of the polished and refined manners of an English dignitary of high birth and station, in whose company we had been.

In his ordinary conversation, there was constantly occurring some appropriate and striking expression. In fact he never expressed himself exactly like other people, and yet without any straining or affectation of effect. No man could have been more conscientiously and sincerely attached to his own Church, both from argument and from those numerous national associations and social feelings which are sometimes more binding even than convictions of reason. He was yet quite free from intolerance and bigotry, and illiberal prejudice. He admired and loved what was great and amiable in those from whom he differed, and differed in many important principles. Thus, as appears from passages I have quoted, he spoke with enthusiasm of the learning and the position of the Church of England. He gloried in the grandeur of her Gothic architecture, as much as any of her own

children could do. On one occasion I recollect his describing, with much interest, a Sunday he passed at Winchester, when a guest of the Bishop, and dilating on the services and "staff of the Cathedral," as he called them; the question was put, evidently expecting an unfavourable reply, "But, Doctor, what did you think of the *chanting*?" His immediate answer was, "Very grand, Sir!" He could discern what was good, and exercise kindness and forbearance towards those from whom he differed far more widely than he did from the Church of England. Thus, when told of a purpose on the part of Roman Catholics to establish in the old town a system of visiting the poor by Sisters of Charity, similar to the visiting in Paris and other continental cities, he exclaimed he was glad to hear it, as it might induce a similar plan of visits from Protestant Sisters of Charity. In his examination before the Committee of the House of Commons respecting his management of St John's, Glasgow, the question was put, "Did you meet with any contradiction on the part of the Roman Catholic clergy of Glasgow?" He replied, "Not in the least; for the clergyman was a party in the negotiation. He attended our meetings, and there was mutual understanding between the clergyman and the members of the committee." (This mutual understanding was, that there should be no attempts on either side at proselytizing, but simply to give education with reading of Scripture. There was this compromise made regarding schools with Roman Catholic children: The Roman Catholic clergyman consented to the use of the Bible as a school-book, according to the authorised version; the Protestants consenting to have Roman Catholic teachers). He had before said to the Committee that he attended at a Roman Catholic school from the delight he had in witnessing the display of native talent among the young Irish, and that he was received with welcome and respect by the Roman Catholic master, who asked him to address the children. Having done so freely, and according to his views, the master thanked him most cordially—and then he added, "This convinced me that a vast deal might be done by kindness, and by discreet and friendly personal intercourse with the Roman Catholics. I may also observe that, whereas it has been alleged that, under the superintendence of a Roman Catholic teacher, there might be a danger of only certain passages of Scripture being read to the exclusion of others, so far as my observation extended, he read quite indiscriminately and impartially over Scripture."

Dr CHALMERS going to the Roman Catholic schools to witness "display of native talent amongst the young Irish," reminds me of a trait in his character not generally perhaps understood, but which was on occasions very marked; I mean his turn for humour and keen sense of the ridiculous. At times he could not control his merriment at ludicrous and grotesque combinations; and I can easily imagine his exquisite enjoyment of answers from the half-naked little Irish urchins. Their odd mixture of acuteness and self-possession, with random confusion of ideas, would be to him irresistibly comic. He had an instinctive sense of the

ludicrous combination of circumstances, and narrated them with great effect. One of the most amusing scenes I remember, was his own description of what happened at Manchester when he had consented to preach a sermon for some public object at a large chapel in that town. He had not been thinking about the matter after he had given his consent to preach; but his eye was attracted by seeing his own name in a printed paper, like an immense play-bill, posted on the walls all about the town. This was a *programme* of the ceremonial for the day. There were to be prayers, anthems, choruses from Handel's Oratorios, and a sermon by the celebrated Dr CHALMERS of Edinburgh! Excessively annoyed at all this display he refused to take any part, or to preach on the occasion. The directors expostulated, and represented what would be the effects of his withdrawal, and the disappointment of the public. The matter was compromised, and Dr CHALMERS was to sit in the vestry till the proper time for him to come out and preach his sermon. But his troubles then only began, for, unfortunately, an anthem, with full instrumental accompaniments, was appointed to follow the sermon. The orchestra being placed immediately behind the pulpit, and more occupied with anticipations of their own performance than with anything else, the musicians annoyed and disturbed the preacher through the whole sermon by their preparations and preliminaries for the grand chorus, "actually," as the Doctor exclaimed, "tuning their very trombones close at my ear before I had finished."

One other feature of mental constitution, and one only I will refer to; and it is an important one, as having its influence not only upon the imagery and ornament of his literary compositions, but, in some instances, upon the general current of his opinion and speculations, and that is his deep admiration of the beautiful in the *material* universe. This admiration was intense, it amounted to a passion, and he evidently had exquisite enjoyment in the contemplation of Nature's works, or rather, I should say, of the goodness and wisdom of the Creator, whether displayed in the wildness or loveliness of natural scenery, the delicate tints and texture of a flower, or the magnificence of the starry heavens. Hence, although no artist himself, he had the greatest interest and enjoyment in the society and conversation of artists. He delighted to hear their remarks on subjects of taste in connection with scenery; on the tints of the landscape, the sky, the ocean, the forms and varieties of clouds, the appearances most suitable for picturesque representation, and the practical rules observed in transferring to the canvas imitations of what is in nature. Hence in his moral reasoning we find all his references, in the way of analogy or illustration, to the beauties and appearances of the natural world, expressed with so much freshness and feeling of reality. He always seems to be impressed with the conviction that, though a fallen world, the fall has chiefly affected the moral and spiritual nature of man himself; that, though the ground was cursed for man's transgression, and so lost the power of supporting the species without toil and labour; yet that, in the *material* world around us,

there remains an impress of primeval beauty,—that there are forms unscathed by the penalties of the primeval curse, and flowers as delicate and fair as those that bloomed in paradise. These sentiments of intense admiration for an external and material world, exercised, I think, considerable influence in modelling his views, and shaping his arguments for *Natural Theology*. He ever delighted in tracing the lineaments of God's moral character in the mirror of the material world, as reflecting his attributes, and as displaying the nature of his handiwork. He deprecated the notion of any essential connection between materialism and sin; and as the abode of man in innocence was a *terrestrial* one, so he believed that in glory there would be provided a new heaven and a new earth, with visible magnificence and material splendour, to be a fitting habitation, and to furnish fitting occupations and enjoyments, for the new and glorified bodies of the redeemed.

I have now, I think, touched upon all those points of character, and all those public acts and deeds, of which I have been capable of forming a judgment, and which have occurred to me as strictly coming within the province of such a paper as the present. In these remarks I have endeavoured to look upon Dr CHALMERS, not as a private friend, but as a public character. I have sought to give a fair transcript of the man as he appeared before us, with no undue partiality arising from those personal feelings of regard and admiration which I am proud to acknowledge. I am certain that those who knew him best esteemed him most. His character bore investigation; and, I think, whatever opinion, in a literary or critical point of view, the world may form of the posthumous volumes, on Scripture Reading, which have been laid before them, it must be allowed that they furnish unequivocal indications of a mind constantly and habitually occupied with sacred things,—of private thoughts and of retired meditations, ever conversant with God and with His holy word.

And now, Sir, to conclude. It will hardly be supposed that I should expect unanimity of opinion in all those questions by which the name of our late distinguished Vice-President has been brought before the notice of his contemporaries. On every subject, indeed, where there are not positive moral precepts or mathematical demonstration, the different tastes and habits of mankind will lead to a difference in their judgments. Different styles of writing, for instance, are congenial with different mental constitutions. The eloquence which affects and even overpowers one man, has little charm or influence over the mind and feelings of another. The early associations of individuals,—the various points of view from which they contemplate the actions of public men, almost inevitably lead to differences in their decisions. In great questions of national or ecclesiastical policy, the conduct utterly condemned by one party, will often be extravagantly lauded by another. It was impossible for any one to take so prominent a position in that

recent movement of our country,—the Disruption of a National Church, with all its accompanying excitements,—its breaking up of old associations,—its contending opinions and hasty sayings,—without running counter to the opinions of many early admirers, without partially, at least, alienating himself from former friends, and separating himself from former coadjutors. On such points it were vain to expect a concurrent judgment on all he has done and said. But of this I feel assured, that none who have had favourable opportunities of personal acquaintance with his character and disposition,—that none who have deeply entered upon a study of his writings, so as fully to appreciate the lofty and benevolent spirit of their sentiments and tendencies, will hesitate to admit that he was both a good and a great man,—that he was imbued with the spirit of Christian philanthropy,—that he had a fervent mind, keen sensibility, and indomitable energy. His highest praise, but, at the same time, his *just* eulogium is, that his fervency of spirit, his sensibility, and his energy, were all exercised and called forth in the one great and magnificent cause,—promoting the glory of God and the welfare of Mankind. In all his meditations, and in all his labours, he had ever distinctly before his eyes the advancement of his fellow-creatures, in their best and truest relations to this world and the world to come.

His greatest delight was to contrive plans and schemes for raising degraded human nature in the scale of moral being,—the favourite object of his contemplation was human nature attaining the highest perfection of which it is capable; and, as that perfection was manifested in saintly individuals, in characters of great acquirement adorned with the graces of Christian piety. His greatest sorrow was to contemplate masses of mankind hopelessly bound to vice and misery by chains of passion, ignorance, and prejudice. As no one more firmly believed in the power of Christianity to regenerate a fallen race,—as faith and experience both conspired to assure him that the only effectual deliverance for the sinful and the degraded was to be wrought by Christian education, and by the active agency of Christian instruction penetrating into the haunts of vice and the abodes of misery;—these acquisitions he strove to gain for all his beloved countrymen; for these he laboured, and for these he was willing to spend and be spent. From the fields of earthly toil and trial he has been removed, and he has entered into his rest. The great business of Christian benevolence, and the contest with ignorance and crime, are left in other hands. But *his* memory will not die, nor his good example in these things be forgotten. His countrymen will do his memory justice. Of the thousands who were assembled to witness the funeral procession which conveyed his earthly remains to the tomb, all felt conviction on that day that a Great Man had fallen in Israel,—that a Scotchman had gone to the grave, of whom Scotland might be proud,—a Scotchman who had earned a name in his country's annals, and a place in his country's literature, which will not pass away.

XXXV.—*On the Theory of Rolling Curves.* By Mr JAMES CLERK MAXWELL.
Communicated by the Rev. Professor KELLAND.

(Read, 19th February 1849.)

There is an important geometrical problem which proposes to find a curve having a given relation to a series of curves described according to a given law. This is the problem of Trajectories in its general form.

The series of curves is obtained from the general equation to a curve by the variation of its parameters. In the general case, this variation may change the form of the curve, but, in the case which we are about to consider, the curve is changed only in position.

This change of position takes place partly by rotation, and partly by transference through space. The rolling of one curve on another is an example of this compound motion.

As examples of the way in which the new curve may be related to the series of curves, we may take the following :—

1. The new curve may cut the series of curves at a given angle. When this angle becomes zero, the curve is the envelope of the series of curves.

2. It may pass through corresponding points in the series of curves. There are many other relations which may be imagined, but we shall confine our attention to this, partly because it affords the means of tracing various curves, and partly on account of the connection which it has with many geometrical problems.

Therefore the subject of this paper will be the consideration of the relations of three curves, one of which is fixed, while the second rolls upon it and traces the third. The subject of rolling curves is by no means a new one. The first idea of the cycloid is attributed to ARISTOTLE, and involutes and evolutes have been long known.

In the "History of the Royal Academy of Sciences" for 1704, page 97, there is a memoir entitled "*Nouvelle formation des Spirales*," by M. VARIGNON, in which he shews how to construct a polar curve from a curve referred to rectangular co-ordinates by substituting the radius vector for the abscissa, and a circular arc for the ordinate. After each curve, he gives the curve into which it is "unrolled," by which he means the curve which the spiral must be rolled upon in order that its pole may trace a straight line; but as this is not the principal subject of his paper, he does not discuss it very fully.

There is also a memoir by M. DE LA HIRE, in the volume for 1706, Part II., page 489, entitled,—"*Methode generale pour réduire toutes les Lignes courbes à des Roulettes, leur generatrice ou leur base étant donnée telle qu'on voudra.*"

M. DE LA HIRE treats curves as if they were polygons, and gives geometrical constructions for finding the fixed curve or the rolling curve, the other two being given; but he does not work any examples.

In the volume for 1707, page 79, there is a paper entitled,—“Methode generale pour déterminer la nature des Courbes formées par le roulement de toutes sortes de Courbes sur une autre Courbe quelconque.” Par M. NICOLE.

M. NICOLE takes the equations of the three curves referred to rectangular co-ordinates, and finds three general equations to connect them. He takes the tracing-point either at the origin of the co-ordinates of the rolled curve or not. He then shews how these equations may be simplified in several particular cases. These cases are,—

- 1st, When the tracing-point is the origin of the rolled curve.
- 2d, When the fixed curve is the same as the rolling curve.
- 3d, When both of these conditions are satisfied.
- 4th, When the fixed line is straight.

He then says, that if we roll a geometric curve on itself, we obtain a new geometric curve, and that we may thus obtain an infinite number of geometric curves.

The examples which he gives of the application of his method are all taken from the cycloid and epicycloid, except one which relates to a parabola, rolling on itself, and tracing a cissoid with its vertex. The reason of so small a number of examples being worked may be, that it is not easy to eliminate the co-ordinates of the fixed and rolling curves from his equations.

The case in which one curve rolling on another produces a circle is treated of in WILLIS'S *Principles of Mechanism*. Class C. *Rolling Contact*.

He employs the same method of finding the one curve from the other which is used here, and he attributes it to EULER (see the *Acta Petropolitana*, vol. v.):

Thus, nearly all the simple cases have been treated of by different authors; but the subject is still far from being exhausted, for the equations have been applied to very few curves, and we may easily obtain new and elegant properties from any curve we please.

Almost all the more notable curves may be thus linked together in a great variety of ways, so that there are scarcely two curves, however dissimilar, between which we cannot form a chain of connected curves.

This will appear in the list of examples given at the end of this paper.

Let there be a curve KAS, whose pole is at C.

Let the angle $DCA = \theta_1$, and $CA = r_1$, and let

$$\theta_1 = \varphi_1(r_1).$$

Let this curve remain fixed to the paper.

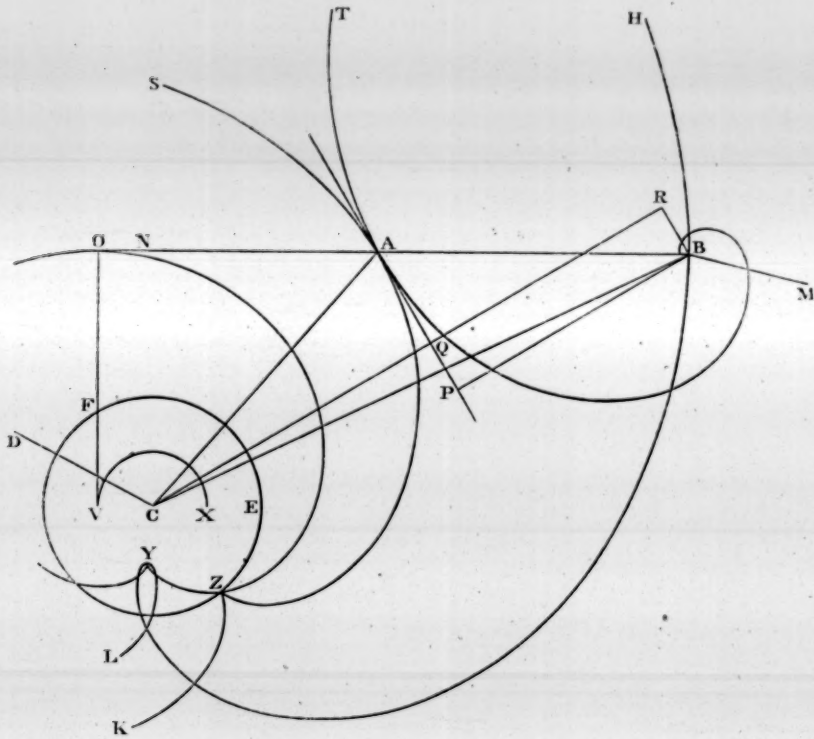
Let there be another curve BAT, whose pole is B.

Let the angle $MBA = \theta_2$, and $BA = r_2$, and let

$$\theta_2 = \varphi_2(r_2).$$

Let this curve roll along the curve KAS without slipping.
 Then the pole B will describe a third curve, whose pole is C.
 Let the angle $DCB = \theta_3$, and $CB = r_3$, and let

$$\theta_3 = \varphi_3(r_3).$$



We have here six unknown quantities, $\theta_1, \theta_2, \theta_3, r_1, r_2, r_3$; but we have only three equations given to connect them, therefore the other three must be sought for in the enunciation.

But before proceeding to the investigation of these three equations, we must premise that the three curves will be denominated as follows:—

The Fixed Curve, Equation, $\theta_1 = \varphi_1(r_1)$

The Rolled Curve, Equation, $\theta_2 = \varphi_2(r_2)$

The Traced Curve, Equation, $\theta_3 = \varphi_3(r_3)$

When it is more convenient to make use of equations between rectangular co-ordinates, we shall use the letters $x_1, y_1, x_2, y_2, x_3, y_3$. We shall always employ

the letters $s_1 s_2 s_3$ to denote the length of the curve from the pole, $p_1 p_2 p_3$ for the perpendiculars from the pole on the tangent, and $q_1 q_2 q_3$ for the intercepted part of the tangent.

Between these quantities, we have the following equations:—

$$\begin{aligned} r &= \sqrt{x^2 + y^2} & \theta &= \tan^{-1} \frac{y}{x} \\ x &= r \cos \theta & y &= r \sin \theta \\ s &= \int \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta & s &= \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \\ p &= \frac{r^2}{\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}} & p &= \frac{y dx - x dy}{\sqrt{(dx)^2 + (dy)^2}} \\ q &= \frac{r \frac{dr}{d\theta}}{\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}} & q &= \frac{x dx + y dy}{\sqrt{(dx)^2 + (dy)^2}} \\ R &= \frac{\left(r^2 + \left(\frac{dr}{d\theta}\right)^2\right)^{\frac{3}{2}}}{r^2 + 2\left(\frac{dr}{d\theta}\right)^2 - r \frac{d^2 r}{d\theta^2}} & R &= \frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}}{\frac{d^2 y}{dx^2}} \end{aligned}$$

We come now to consider the three equations of rolling which are involved in the enunciation. Since the second curve rolls upon the first *without slipping*, the length of the fixed curve at the point of contact is the measure of the length of the rolled curve, therefore we have the following equation to connect the fixed curve and the rolled curve,—

$$s_1 = s_2$$

Now, by combining this equation with the two equations

$$\left\{ \begin{array}{l} \theta_1 = \varphi_1(r_1) \\ \theta_2 = \varphi_2(r_2) \end{array} \right\} \text{ or } \left\{ \begin{array}{l} x_1 = \psi_1(y_1) \\ x_2 = \psi_2(y_2) \end{array} \right\},$$

it is evident that from any of the four quantities $\theta_1 r_1 \theta_2 r_2$ or $x_1 y_1 x_2 y_2$, we can obtain the other three, therefore we may consider these quantities as known functions of each other.

Since the curve *rolls* on the fixed curve, they must have a common tangent.

Let PA be this tangent, draw BP, CQ perpendicular to PA, produce CQ, and draw BR perpendicular to it, then we have CA = r_1 , BA = r_2 , and CB = r_3 ; CQ = p_1 , PB = p_2 , and BN = p_3 ; AQ = q_1 , AP = q_2 and CN = q_3 .

Also,

$$\begin{aligned} r_3^2 &= CB^2 = CR^2 + RB^2 = (CQ + PB)^2 + (AP - AQ)^2 \\ &= (p_1 + p_2)^2 + (q_2 - q_1)^2 \\ &= p_1^2 + 2 p_1 p_2 + p_2^2 + r_2^2 - p_2^2 - 2 q_1 q_2 + r_1^2 - p_1^2 \\ r_3^2 &= r_1^2 + r_2^2 + 2 p_1 p_2 - 2 q_1 q_2 \end{aligned}$$

Since the first curve is fixed to the paper, we may find the angle θ_1

Thus $\theta_3 = DCB = DCA + ACQ + RCB$

$$= \theta_1 + \tan^{-1} \frac{q_1}{p_1} + \tan^{-1} \frac{RB}{RC}$$

$$\theta_3 = \theta_1 + \tan^{-1} \frac{d r_1}{r_1 d \theta_1} + \tan^{-1} \frac{q_2 - q_1}{p_2 + p_1}$$

Thus we have found three independent equations, which, together with the equations of the curves, make up six equations, of which each may be deduced from the others. There is an equation connecting the radii of curvature of the three curves which is sometimes of use.

The angle through which the rolled curve revolves during the description of the element $d s_3$, is equal to the angle of contact of the fixed curve and the rolling curve, or to the sum of their curvatures,

$$\therefore \frac{d s_3}{r_2} = \frac{d s_1}{R_1} + \frac{d s_2}{R_2}$$

But the radius of the rolled curve has revolved in the opposite direction through an angle equal to $d \theta_2$, therefore the angle between two successive positions of r_2 is equal to $\frac{d s_3}{r_2} - d \theta_2$. Now this angle is the angle between two successive positions of the normal to the traced curve, therefore, if O be the centre of curvature of the traced curve, it is the angle which $d s_3$ or $d s_1$ subtends at O. Let OA = T, then

$$\begin{aligned} \frac{d s_3}{R_3} &= \frac{r_2 d \theta_2}{T} = \frac{d s_3}{r_2} - d \theta_2 = \frac{d s_2}{R_1} + \frac{d s_2}{R_2} - d \theta_2 \\ \therefore r_2 \frac{d \theta_2}{d s_2} \frac{1}{T} &= \frac{1}{R_1} + \frac{1}{R_2} - r_2 \left(\frac{d \theta_2}{d s_2} \right)^2 \\ \therefore \frac{p_2}{r_2} \left(\frac{1}{T} + \frac{1}{r_2} \right) &= \frac{1}{R_1} + \frac{1}{R_2} \end{aligned}$$

As an example of the use of this equation, we may examine a property of the logarithmic spiral.

In this curve, $p = m r$, and $R = \frac{r}{m}$, therefore if the rolled curve be the logarithmic spiral

$$m \left(\frac{1}{T} + \frac{1}{r_2} \right) = \frac{1}{R_1} + \frac{m}{r_2}$$

$$\frac{m}{T} = \frac{1}{R_1}$$

therefore AO in the figure = $m R_1$, and $\frac{AO}{R_1} = m$.

Let the locus of O, or the evolute of the traced curve LYBH, be the curve OZY, and let the evolute of the fixed curve KZAS be FEZ, and let us consider FEZ as the fixed curve, and OZY as the traced curve.

Then in the triangles BPA, AOF, we have OAF = PBA, and $\frac{OA}{AF} = m = \frac{BP}{AB}$,

therefore the triangles are similar, and FOA = APB = $\frac{\pi}{2}$, therefore OF is perpendicular to OA, the tangent to the curve OZY, therefore OF is the radius of the curve which when rolled on FEZ traces OZY, and the angle which the curve makes with this radius is OFA = PAB = $\sin^{-1} m$, which is constant, therefore the curve, which, when rolled on FEZ, traces OZY, is the logarithmic spiral. Thus we have proved the following proposition: "The involute of the curve traced by the pole of a logarithmic spiral which rolls upon any curve, is the curve traced by the pole of the same logarithmic spiral when rolled on the involute of the primary curve."

It follows from this, that if we roll on any curve a curve having the property $p_1 = m_1 r_1$, and roll another curve having $p_2 = m_2 r_2$ on the curve traced, and so on, it is immaterial in what order we roll these curves. Thus, if we roll a logarithmic spiral, in which $p = m r$, on the n th involute of a circle whose radius is a , the curve traced is the $n + 1$ th involute of a circle whose radius is $a \sqrt{1 - m^2}$.

Or, if we roll successively m logarithmic spirals, the resulting curve is the $n + m$ th involute of a circle, whose radius is

$$a \sqrt{1 - m_1^2} \sqrt{1 - m_2^2} \sqrt{\text{etc.}}$$

We now proceed to the cases in which the solution of the problem may be simplified. This simplification is generally effected by the consideration that the radius vector of the rolled curve is the normal drawn from the traced curve to the fixed curve.

In the case in which the curve is rolled on a straight line, the perpendicular on the tangent of the rolled curve is the distance of the tracing point from the straight line; therefore, if the traced curve be defined by an equation in x_3 and y_3 .

$$x_3 = p_2 = \frac{r_2^2}{\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}} \quad \dots \quad (1.)$$

and

$$r_2 = x \sqrt{\left(\frac{dx}{dy}\right)^2 + 1} \quad \dots \quad (2.)$$

By substituting for r_2 in the first equation, its value, as derived from the second, we obtain

$$x^2 \left(\frac{dx_3}{dy_3}\right)^2 \left[\left(\frac{dx_3}{dy_3}\right)^2 + 1 \right] = \left(\frac{dr_2}{d\theta_2}\right)^2$$

If we know the equation to the rolled curve, we may find $\left(\frac{dr_2}{d\theta_2}\right)^2$ in terms of r_2 , then by substituting for r_2 its value in the second equation, we have an equation containing x_3 and $\frac{dx_3}{dy_3}$, from which we find the value of $\frac{dx_3}{dy_3}$ in terms of x_3 , the integration of this gives the equation of the traced curve.

As an example, we may find the curve traced by the pole of a hyperbolic spiral which rolls on a straight line.

The equation of the rolled curve is $\theta_2 = \frac{a}{r_2}$

$$\therefore \left(\frac{dr_2}{d\theta_2}\right)^2 = \frac{r_2^4}{a^2}$$

$$= x_3^2 \left(\frac{dx_3}{dy_3}\right)^2 \left[\left(\frac{dx_3}{dy_3}\right)^2 + 1 \right] = \frac{x_3^4}{a^2} \left[\left(\frac{dx_3}{dy_3}\right)^2 + 1 \right]^2$$

$$\therefore a^2 \left(\frac{dx_3}{dy_3}\right)^2 = x_3^2 \left[\left(\frac{dx_3}{dy_3}\right)^2 + 1 \right]$$

$$\therefore \frac{dx_3}{dy_3} = \frac{x_3}{\sqrt{a^2 - x_3^2}}$$

This is the differential equation of the tractory of the straight line, which is the curve traced by the pole of the hyperbolic spiral.

By eliminating x_3 in the two equations, we obtain

$$\frac{dr_2}{d\theta_2} = r_2 \left(\frac{dx_3}{dy_3}\right)$$

This equation serves to determine the rolled curve when the traced curve is given.

As an example we shall find the curve, which being rolled on a straight line, traces a common catenary.

Let the equation to the catenary be

$$x = \frac{a}{2} \left(e^{\frac{y}{a}} + e^{-\frac{y}{a}} \right)$$

Then

$$\frac{dx_3}{dy_3} = \sqrt{\frac{x_3^2}{a^2} - 1}$$

$$\begin{aligned}
 \therefore \left(\frac{d r_2}{d \theta_2} \right)^2 &= \frac{r_2^2 \cdot r^4}{a^2 \left(\frac{d r_2}{d \theta_2} \right)^2 + r_2^2} - r_2^2 \\
 \therefore \left[\left(\frac{d r_2}{d \theta_2} \right)^2 + r_2^2 \right]^2 &= \left(\frac{r_2^2}{a} \right)^2 \\
 \therefore \left(\frac{d r_2}{d \theta_2} \right)^2 &= \frac{r_2^2}{a} (r - a) \\
 \therefore \frac{d \theta}{d r} &= \frac{1}{r \sqrt{\frac{r}{a} - 1}} \quad \text{then by integration} \\
 \theta &= \cos^{-1} \left(\frac{2a}{r} - 1 \right) \\
 r &= \frac{2a}{1 + \cos \theta}
 \end{aligned}$$

This is the polar equation of the parabola, the focus being the pole, therefore, if we roll a parabola on a straight line, its focus will trace a catenary.

The rectangular equation of this parabola is $x^2 = 4ay$, and we shall now consider what curve must be rolled along the axis of y to trace the parabola.

By the second equation (2.),

$$\begin{aligned}
 r_2 &= x_3 \sqrt{\frac{4a^2}{x_3^2} + 1} \quad \text{but } x_3 = p_2 \\
 \therefore r_2 &= \sqrt{4a^2 + p_2^2} \\
 \therefore r_2^2 &= p_2^2 = 4a^2 \\
 \therefore 2a &= \sqrt{r_2^2 - p_2^2} = q_2
 \end{aligned}$$

but q_2 is the perpendicular on the normal, therefore the normal to the curve always touches a circle whose radius is Qa , therefore the curve is the involute of this circle.

Therefore we have the following method of describing a catenary by continued motion.

Describe a circle whose radius is twice the parameter of the catenary; roll a straight line on this circle, then any point in the line will describe an involute of the circle; roll this curve on a straight line, and the centre of the circle will describe a parabola; roll this parabola on a straight line, and its focus will trace the catenary required.

We come now to the case in which a straight line rolls on a curve.

When the tracing-point is in the straight line, the problem becomes that of involutes and evolutes, which we need not enter upon, and when the tracing-point is

not in the straight line, the calculation is somewhat complex, we shall therefore consider only the relations between the curves described in the first and second cases.

Definition.—The curve which cuts at a given angle all the circles of a given radius whose centres are in a given curve, is called a tractory of the given curve.

Let a straight line roll on a curve A, and let a point in the straight line describe a curve B, and let another point, whose distance from the first point is b , and from the straight line a , describe a curve C, then it is evident that the curve B cuts the circle whose centre is in C, and whose radius is b , at an angle whose sine is equal to $\frac{a}{b}$, therefore the curve B is a tractory of the curve C.

When $a = b$, the curve B is the orthogonal tractory of the curve C. If tangents equal to a be drawn to the curve B, they will be terminated in the curve C; and if one end of a thread be carried along the curve C, the other end will trace the curve B.

When $a = 0$, the curves B and C are both involutes of the curve A, they are always equidistant from each other, and if a circle, whose radius is b , be rolled on the one, its centre will trace the other.

If the curve A is such that, if the distance between two points measured along the curve is equal to b , the two points are similarly situate, then the curve B is the same with the curve C. Thus, the curve A may be a re-entrant curve, the circumference of which is equal to b .

When the curve A is a circle, the curves B and C are always the same.

The equations between the radii of curvature become

$$\frac{1}{T} + \frac{1}{r_2} = \frac{r}{a R_1}$$

When $a = 0$, $T = 0$, or the centre of curvature of the curve B is at the point of contact. Now, the normal to the curve C passes through this point, therefore—

“The normal to any curve passes through the centre of curvature of its tractory.”

In the next case, one curve, by rolling on another, produces a straight line. Let this straight line be the axis of y , then, since the radius of the rolled curve is perpendicular to it, and terminates in the fixed curve, and since these curves have a common tangent, we have these equations,

$$r_2 = x_1 \frac{dy_1}{dx_1} = r_2 \frac{d\theta_2}{dr_2}$$

If the equation of the rolled curve be given, find $\frac{d\theta_2}{dr_2}$ in terms of r_2 , substitute

x_1 for r_2 , and multiply by x_1 , equate the result to $\frac{dy}{dx}$, and integrate.

Thus, if the equation of the rolled curve be

$$\theta = A r^{-n} + \text{etc.} + K r^{-2} + L r^{-1} + M \log r + N r + \text{etc.} + Z r^n$$

$$\frac{d\theta}{dr} = -n A r^{-(n+1)} - \text{etc.} - 2 K r^{-3} - L r^{-2} + M r^{-1} + N + \text{etc.} + n Z r^{n-1}$$

$$\frac{dy}{dx} = -n A x^{-n} - \text{etc.} - 2 K x^{-2} - L x^{-1} + M + N x + \text{etc.} + n Z x^n$$

$$y = \frac{n}{n-1} A x^{1-n} + \text{etc.} + 2 K x^{-1} - L \log x + M x + \frac{1}{2} N x^2 + \text{etc.} + \frac{n}{n+1} Z x^{n+1}$$

which is the equation of the fixed curve.

If the equation of the fixed curve be given, find $\frac{dy}{dx}$ in terms of x , substitute r for x , and divide by r , equate the result to $\frac{d\theta}{dr}$, and integrate.

Thus, if the fixed curve be the orthogonal tractory of the straight line, whose equation is

$$y = a \log \frac{x}{a + \sqrt{a^2 - x^2}} + \sqrt{a^2 - x^2}$$

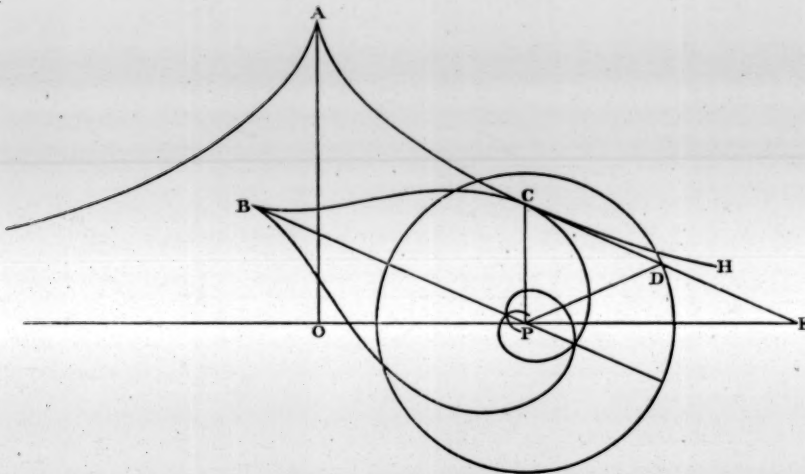
$$\frac{dy}{dx} = \frac{\sqrt{a^2 - x^2}}{x}$$

$$\frac{d\theta}{dr} = \frac{\sqrt{a^2 - r^2}}{r^2}$$

$$\theta = \cos^{-1} \frac{r}{a} - \sqrt{\frac{a^2}{r^2} - 1}$$

this is the equation to the orthogonal tractory of a circle whose diameter is equal to the constant tangent of the fixed curve, and its constant tangent equal to half that of the fixed curve.

This property of the tractory of the circle may be proved geometrically, thus—Let P be the centre of a circle whose radius is PD, and let CD be a line constantly equal to the radius. Let BCP be the curve described by the point C when the point D is moved along the circumference of the circle, then if tangents equal to CD be drawn to the curve, their extremities will be in the circle. Let ACH be the curve on which BCP rolls, and let OPE be the straight line traced by the pole, let CDE be the common tangent, let it cut the circle in D, and the straight line in E.



Then $CD = PD \therefore \angle DCP = \angle DPC$, and CP is perpendicular to OE ,
 $\therefore \angle CPE = \angle DCP + \angle DEP$. Take away $\angle DCP = \angle DPC$, and there remains
 $\angle DPE = \angle DEP \therefore PD = DE \therefore CE = 2 PD$.

Therefore the curve ACH has a constant tangent equal to the diameter of the circle, therefore ACH is the orthogonal tractrix of the straight line, which is the tractrix or equitangential curve.

The operation of finding the fixed curve from the rolled curve is what Sir JOHN LESLIE calls "divesting a curve of its radiated structure."

The method of finding the curve which must be rolled on a circle to trace a given curve is mentioned here because it generally leads to a double result, for the normal to the traced curve cuts the circle in two points, either of which may be a point in the rolled curve.

Thus, if the traced curve be the involute of a circle concentric with the given circle, the rolled curve is one of two similar logarithmic spirals.

If the line traced be a tangent to the circle, the rolled curve is either of the parts of the polar catenary.

If the curve traced be the spiral of ARCHIMEDES, the rolled curve may be either the hyperbolic spiral or the straight line.

In the next case, one curve rolls on another and traces a circle.

Since the curve traced is a circle, the distance between the poles of the fixed curve and the rolled curve is always the same; therefore, if we fix the rolled curve and roll the fixed curve, the curve traced will still be a circle, and, if we fix the poles of both the curves, we may roll them on each other without friction.

Let a be the radius of the traced circle, then the sum or difference of the radii

of the other curves is equal to a , and the angles which they make with the radius at the point of contact are equal.

$$\therefore r_1 = \pm (a \pm r_2) \text{ and } r_1 \frac{d\theta_1}{dr_1} = r_2 \frac{d\theta_2}{dr_2}$$

$$\therefore \frac{d\theta_2}{dr_2} = \frac{\pm (a \pm r_2)}{r_2} \frac{d\theta_1}{dr_1}$$

If we know the equation between θ_1 and r_1 , we may find $\frac{d\theta_1}{dr_1}$ in terms of r_1 , substitute $\pm (a \pm r_2)$ for r_1 , multiply by $\frac{\pm (a \pm r_2)}{r_2}$, and integrate.

Thus, if the equation between θ_1 and r_1 be

$$r_1 = a \sec \theta_1$$

which is the polar equation of a straight line touching the traced circle whose equation is $r = a$,

then

$$\begin{aligned} \frac{d\theta}{dr_1} &= \frac{a}{r_1 \sqrt{r_1^2 - a^2}} \\ &= \frac{a}{(r_2 \pm a) \sqrt{r_2^2 \pm 2r_2 a}} \\ \frac{d\theta_2}{dr_2} &= \frac{r_2 \pm a}{r_2} \frac{a}{(r_2 \pm a) \sqrt{r_2^2 \pm 2r_2 a}} \\ &= \frac{a}{r_2 \sqrt{r_2^2 \pm 2r_2 a}} \\ \theta_2 &= \pm \sqrt{1 \pm 2\frac{a}{r}} \\ r_2 &= \frac{2a}{\theta_2^2 - 1} = \frac{2a}{\theta^2 - 1} \end{aligned}$$

Now, since the rolling curve is a straight line, and the tracing point is not in its direction, we may apply to this example the observations which have been made upon tractories.

Let, therefore, the curve $r = \frac{2a}{\theta^2 - 1}$ be denoted by A, its involute by B, and the circle traced by C, then B is the tractory of C; therefore the involute of the curve $r = \frac{2a}{\theta^2 - 1}$ is the tractory of the circle, the equation of which is $\theta = \cos^{-1} \frac{r}{a} - \sqrt{\frac{a^2}{r^2} - 1}$. The curve whose equation is $r = \frac{2a}{\theta^2 - 1}$ seems to be among spirals what the catenary is among curves whose equations are between rectangular co-ordinates; for, if we represent the vertical direction by the radius

vector, the tangent of the angle which the curve makes with this line is proportional to the length of the curve reckoned from the origin; the point at the distance a from a straight line rolled on this curve generates a circle, and when rolled on the catenary produces a straight line; the involute of this curve is the tractory of the circle, and that of the catenary is the tractory of the straight line, and the tractory of the circle rolled on that of the straight line traces the straight line; if this curve is rolled on the catenary, it produces the straight line touching the catenary at its vertex; the method of drawing tangents is the same as in the catenary, namely, by describing a circle whose radius is a on the production of the radius vector, and drawing a tangent to the circle from the given point.

In the next case, the rolled curve is the same as the fixed curve. It is evident that the traced curve will be similar to the locus of the intersection of the tangent with the perpendicular from the pole; the magnitude, however, of the traced curve will be double that of the other curve; therefore, if we call $r_0 = \phi_0, \theta_0$ the equation to the fixed curve, $r_1 = \phi_1, \theta_1$ that of the traced curve, we have,

$$r_1 = 2p_0 \quad \theta_1 = \theta_0 - \cos^{-1} \frac{p_0}{r_0} = \theta_0 - \frac{\pi}{2} + \sin^{-1} \frac{p_0}{r_0}$$

$$\text{also, } \frac{p_1}{r_1} = \frac{p_0}{r_0}$$

$$\text{Similarly, } r_2 = 2p_1 = 2r_1 \frac{p_0}{r_0} = 4 \frac{p_0^2}{r_0} 4r_0 \left(\frac{p_0}{r_0}\right)^2, \quad \theta^2 = \theta_0 - 2 \cos^{-1} \frac{p_0}{r_0}$$

$$\text{Similarly, } r_n = 2p_{n-1} = 2r_{n-1} \frac{p_0}{r_0} \text{ etc.} = 2^n r_0 \left(\frac{p_0}{r_0}\right)^n$$

$$\text{and } \frac{p_n}{r_n} = \frac{p_0}{r_0}$$

$$\theta_n = \theta_0 - n \cos^{-1} \frac{p_0}{r_0}$$

$$\theta_n = \theta_0 - n \cos^{-1} \frac{p_n}{r_n}$$

Let θ_n become θ_n^1 ; θ_0, θ_0^1 and $\frac{p_0}{r_0}, \frac{p_0^1}{r_0^1}$. Let $\theta_n^1 - \theta_n = \alpha$

$$\theta_n^1 = \theta_0^1 - n \cos^{-1} \frac{p_0^1}{r_0^1}$$

$$\alpha = \theta_n^1 - \theta_n = \theta_0^1 - \theta_0 - n \cos^{-1} \frac{p_n^1}{r_n^1} + n \cos^{-1} \frac{p_n}{r_n}$$

$$\therefore \cos^{-1} \frac{p_n}{r_n} - \cos^{-1} \frac{p_n^1}{r_n^1} = \frac{\alpha}{n} + \frac{\theta_0 - \theta_0^1}{n}$$

Now, $\cos^{-1} \frac{p_n}{r_n}$ is the complement of the angle at which the curve cuts the radius

vector, and $\cos^{-1} \frac{p_n}{r_n} - \cos^{-1} \frac{p^1_n}{r^1_n}$ is the variation of this angle when θ_n varies by an angle equal to α . Let this variation = ϕ ; then if $\theta_0 - \theta_0^1 = \beta$

$$\phi = \frac{\alpha}{n} + \frac{\beta}{n}$$

Now, if n increases, ϕ will diminish; and if n become infinite,

$$\phi = \frac{\alpha}{\infty} + \frac{\beta}{\infty} = 0 \text{ when } \alpha \text{ and } \beta \text{ are finite.}$$

Therefore, when n is infinite, ϕ vanishes; therefore, the curve cuts the radius vector at a constant angle; therefore the curve is the logarithmic spiral.

Therefore, if any curve be rolled on itself, and the operation repeated an infinite number of times, the resulting curve is the logarithmic spiral.

Hence we may find, analytically, the curve which, being rolled on itself, traces itself.

For the curve which has this property, if rolled on itself, and the operation repeated an infinite number of times, will still trace itself.

But, by this proposition, the resulting curve is the logarithmic spiral; therefore the curve required is the logarithmic spiral. As an example of a curve rolling on itself, we will take the curve whose equation is

$$r_0 = 2^n a \left(\cos \frac{\theta_0}{n} \right)^n$$

$$\text{Here } -\frac{d r_0}{d \theta^0} = 2^n a \left(\sin \frac{\theta_0}{n} \right) \left(\cos \frac{\theta_0}{n} \right)^{n-1}$$

$$\therefore r_1 = 2 p_0 = 2 \frac{2^{2n} a^2 \left(\cos \frac{\theta_0}{n} \right)^{2n}}{\sqrt{2^{2n} a^2 \left(\cos \frac{\theta_0}{n} \right)^{2n} + 2^{2n} a^2 \left(\sin \frac{\theta_0}{n} \right)^2 \left(\cos \frac{\theta_0}{n} \right)^{2n-2}}}$$

$$r_1 = 2 \frac{2^n a \left(\cos \frac{\theta_0}{n} \right)^{n+1}}{\sqrt{\left(\cos \frac{\theta_0}{n} \right)^2 + \left(\sin \frac{\theta_0}{n} \right)^2}} = 2^{n+1} a \left(\cos \frac{\theta_0}{n} \right)^{n+1}$$

$$\text{Now } \theta_1 - \theta_0 = -\cos^{-1} \frac{p_0}{r_0} = -\cos^{-1} \cos \frac{\theta_0}{n} = \frac{\theta_0}{n}$$

$$\therefore \theta_0 = \theta_1 \frac{n}{n+1}$$

substituting this value of θ_0 in the expression for r_1

$$r_1 = 2^{n+1} a \left(\cos \frac{\theta_1}{n+1} \right)^{n+1}$$

similarly if the operation be repeated m times, the resulting curve is

$$r_m = 2^{n+m} a \left(\cos \frac{\theta_m}{n+m} \right)^{n+m}$$

When $n = 1$, the curve is

$$r = 2 a \cos \theta$$

the equation to a circle, the pole being in the circumference.

When $n = 2$, it is the equation to the cardioid.

$$r = 4 a \left(\cos \frac{\theta}{2} \right)^2$$

In order to obtain the cardioid from the circle, we roll the circle upon itself, and thus obtain it by one operation; but there is an operation which, being performed on a circle, and again on the resulting curve, will produce a cardioid, and the intermediate curve between the circle and cardioid is

$$r = 2^{\frac{3}{2}} a \left(\cos \frac{2\theta}{3} \right)^{\frac{3}{2}}$$

As the operation of rolling a curve on itself is represented by changing n into $\frac{n+1}{n+\frac{1}{2}}$ in the equation, so this operation may be represented by changing n into $\frac{n+1}{n+\frac{1}{2}}$.

Similarly there may be many other fractional operations performed upon the curves comprehended under the equation

$$r = 2^n a \left(\cos \frac{\theta}{n} \right)^n$$

We may also find the curve, which, being rolled on itself, will produce a given curve, by making $n = -1$.

We may likewise prove by the same method as before, that the result of performing this inverse operation an infinite number of times is the logarithmic spiral.

As an example of the inverse method, let the traced line be straight, let its equation be

$$r_0 = 2 a \sec \theta_0$$

$$\text{then } \frac{p_{-1}}{r_{-1}} = \frac{p_0}{r_0} = \frac{2a}{r_0} = \frac{2a}{2p_{-1}}$$

$$\therefore p_{-1}^2 = a r_{-1}$$

therefore suppressing the suffix,

$$\begin{aligned}\frac{r^4}{r^2 + \frac{d}{d\theta} r^2} &= a r \\ \therefore \left(\frac{dr}{d\theta}\right)^2 &= \frac{r^3}{a} - r^2 \\ \therefore \frac{d\theta}{dn} &= \frac{1}{r \sqrt{\frac{r}{a} - 1}} \\ \therefore \theta &= \cos^{-1} \left(\frac{2a}{r} - 1 \right) \\ r &= \frac{2a}{1 - \cos \theta}\end{aligned}$$

the polar equation of the parabola whose parameter is $4a$.

The last case which we shall here consider, affords the means of constructing two wheels whose centres are fixed, and which shall roll on each other, so that the angle described by the first shall be a given function of the angle described by the second.

$$\text{Let } \theta_2 = \varphi \theta_1 \text{ then } r_1 + r_2 = a, \text{ and } \frac{d\theta_2}{d\theta_1} = \frac{r}{r_2}$$

$$\therefore \frac{d(\varphi \theta_1)}{d\theta_1} = \frac{r_1}{a - r_1}$$

Let us take as an example, the pair of wheels which will represent the angular motion of a comet in a parabola.

$$\text{Here } \theta_2 = \tan \frac{\theta_1}{2} \quad \therefore \frac{d\theta_2}{d\theta_1} = \frac{1}{\cos^2 \frac{\theta_1}{2}} = \frac{r}{a - r_1}$$

$$\therefore \frac{r_1}{a} = \frac{1}{2 + \cos \theta_1}$$

therefore the first wheel is an ellipse, whose major axis is equal to $\frac{4}{3}$ of the distance between the centres of the wheels, and in which the distance between the foci is half the major axis.

Now, since

$$\theta_1 = 2 \tan^{-1} \theta_2 \text{ and } r_1 = a - r_2$$

$$\frac{r}{a} = 1 + \frac{1}{2(2 - \theta^4)}$$

$$\theta^4 = 2 - \frac{1}{\frac{2}{r} - 2}$$

which is the equation to the wheel which revolves with constant angular velocity.

Before proceeding to give a list of examples of rolling curves, we shall state a theorem which is almost self-evident after what has been shewn previously.

Let there be three curves, A, B, and C. Let the curve A, when rolled on itself, produce the curve B, and when rolled on a straight line let it produce the curve C, then, if the dimensions of C be doubled, and B be rolled on it, it will trace a straight line.

A Collection of Examples of Rolling Curves.

1st. Examples of a curve rolling on a straight line.

Ex. 1. When the rolling curve is a circle whose tracing-point is in the circumference, the curve traced is a cycloid, and when the point is not in the circumference, the cycloid becomes a trochoid.

Ex. 2. When the rolling curve is the involute of the circle whose radius is $2a$, the traced curve is a parabola whose parameter is $4a$.

Ex. 3. When the rolled curve is the parabola whose parameter is $4a$, the traced curve is a catenary whose parameter is a , and whose vertex is distant a from the straight line.

Ex. 4. When the rolled curve is a logarithmic spiral, the pole traces a straight line which cuts the fixed line at the same angle as the spiral cuts the radius vector.

Ex. 5. When the rolled curve is the hyperbolic spiral, the traced curve is the tractory of the straight line.

Ex. 6. When the rolled curve is the polar catenary

$$\theta = \pm \sqrt{1 \pm \frac{2a}{r}}$$

the traced curve is a circle whose radius is a , and which touches the straight line.

Ex. 7. When the equation of the rolled curve is

$$\theta = \log \left(\sqrt{\frac{r^4}{a^4} - 1} + \frac{r^2}{a^2} \right) - \log \left(\sqrt{\frac{a^4}{r^4} + 1} - \frac{a^2}{r^2} \right)$$

the traced curve is the hyperbola whose equation is

$$y^2 = a^2 + x^2$$

2d. In the examples of a straight line rolling on a curve, we shall use the letters A, B, and C to denote the three curves treated of in page 555.

Ex. 1. When the curve A is a circle whose radius is a , then the curve B is the involute of that circle, and the curve C is the spiral of Archimedes, $r = a\theta$.

Ex. 2. When the curve A is a catenary whose equation is

$$x = \frac{a}{2} \left(e^{\frac{y}{a}} + e^{-\frac{y}{a}} \right)$$

the curve B is the tractory of the straight line, whose equation is

$$y = a \log \frac{x}{a + \sqrt{a^2 - x^2}} + \sqrt{a^2 - x^2}$$

and C is a straight line at a distance a from the vertex of the catenary.

Ex. 3. When the curve A is the polar catenary

$$\theta = \pm \sqrt{1 \pm \frac{2a}{r}}$$

the curve B is the tractory of the circle

$$\theta = \cos^{-1} \frac{r}{a} - \sqrt{\frac{a^2}{r^2} - 1}$$

and the curve C is a circle of which the radius is $\frac{a}{2}$.

3d. Examples of one curve rolling on another, and tracing a straight line.

Ex. 1. The curve whose equation is

$$\theta = A r^{-n} + \text{etc.} + K r^{-2} + L r^{-1} + M \log r + N r + \text{etc.} + Z r^r$$

when rolled on the curve whose equation is

$$y = \frac{n}{n-1} A x^{1-n} + \text{etc.} + 2 K x^{-1} - L \log x + M x + \frac{1}{2} N x^2 + \text{etc.} + \frac{n}{n+1} Z x^{n+1}$$

traces the axis of y .

Ex. 2. The circle whose equation is $r = a \cos \theta$ rolled on the circle whose radius is a traces a diameter of the circle.

Ex. 3. The curve whose equation is

$$\theta = \sqrt{\frac{2a}{r} - 1} - \text{versin}^{-1} \frac{r}{a}$$

rolled on the circle whose radius is a traces the tangent to the circle.

Ex. 4. If the fixed curve be a parabola whose parameter is $4a$, and if we roll on it the spiral of Archimedes $r = a\theta$, the pole will trace the axis of the parabola.

Ex. 5. If we roll an equal parabola on it, the focus will trace the directrix of the first parabola.

Ex. 6. If we roll on it the curve $r = \frac{a}{4\theta^2}$ the pole will trace the tangent at the vertex of the parabola.

Ex. 7. If we roll the curve whose equation is

$$r = a \cos \left(\frac{a}{b} \theta \right)$$

on the ellipse whose equation is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

the pole will trace the axis b .

Ex. 8. If we roll the curve whose equation is

$$r = \frac{a}{2} \left(e^{\frac{a\theta}{b}} - e^{-\frac{a\theta}{b}} \right)$$

on the hyperbola whose equation is

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$

the pole will trace the axis b .

Ex. 9. If we roll the lituus, whose equation is

$$r^2 = \frac{a^2}{3\theta}$$

on the hyperbola whose equation is

$$xy = a^2$$

the pole will trace the asymptote.

Ex. 10. The cardioid whose equation is

$$r = a(1 + \cos \theta)$$

rolled on the cycloid whose equation is

$$y = a \operatorname{versin}^{-1} \frac{x}{a} + \sqrt{2ax - x^2}$$

traces the base of the cycloid.

Ex. 11. The curve whose equation is

$$\theta = \operatorname{versin}^{-1} \frac{r}{a} + 2 \sqrt{\frac{2a}{r} - 1}$$

rolled on the cycloid traces the tangent at the vertex.

Ex. 12. The straight line whose equation is

$$r = a \sec \theta$$

rolled on a catenary whose parameter is a , traces a line whose distance from the vertex is a .

Ex. 13. The part of the polar catenary whose equation is

$$\theta = \pm \sqrt{1 + \frac{2a}{r}}$$

rolled on the catenary traces the tangent at the vertex.

Ex. 14. The other part of the polar catenary whose equation is

$$\theta = \pm \sqrt{1 - \frac{2a}{r}}$$

rolled on the catenary traces a line whose distance from the vertex is equal to $2a$.

Ex. 15. The tractory of the circle whose diameter is a , rolled on the tractory of the straight line whose constant tangent is a , produces the straight line.

Ex. 16. The hyperbolic spiral whose equation is

$$r = \frac{a}{\theta}$$

rolled on the logarithmic curve whose equation is

$$y = a \log \frac{x}{a}$$

traces the axis of y or the asymptote.

Ex. 17. The involute of the circle whose radius is a , rolled on an orthogonal trajectory of the catenary whose equation is

$$y = \frac{x}{2a} \sqrt{x^2 - a^2} + \frac{a}{2} \log \left(\sqrt{\frac{x^2}{a^2} - 1} + \frac{x}{a} \right)$$

traces the axis of y .

Ex. 18. The curve whose equation is

$$\theta = \left(\frac{a}{x} + 1 \right) \sqrt{2 \frac{a}{x} + 1}$$

rolled on the witch, whose equation is

$$y = 2a \sqrt{\frac{2a}{x} - 1}$$

traces the asymptote.

Ex. 19. The curve whose equation is

$$r = a \tan \theta$$

rolled on the curve whose equation is

$$y = \frac{a}{2} \log \left(\frac{x^2}{a^2} - 1 \right)$$

traces the axis of y .

Ex. 20. The curve whose equation is

$$\theta = \frac{2r}{\sqrt{a^2 - r^2}}$$

rolled on the curve whose equation is

$$y = \frac{x^2}{\sqrt{a^2 - x^2}} \quad \text{or } r = a \tan \theta$$

traces the axis of y .

Ex. 21. The curve whose equation is

$$r = a (\sec \theta - \tan \theta)$$

rolled on the curve whose equation is

$$y = a \log \left(\frac{x^2}{a^2} + 1 \right)$$

traces the axis of y .

4th. Examples of pairs of rolling curves which have their poles at a fixed distance = a

- Ex. 1. $\left\{ \begin{array}{l} \text{The straight line whose equation is} \\ \text{The polar catenary whose equation is} \end{array} \right. \quad \begin{array}{l} \theta = \sec^{-1} \frac{r}{a} \\ \theta = \pm \sqrt{1 \pm \frac{2a}{r}} \end{array}$
- Ex. 2. Two equal ellipses or hyperbolas centred at the foci.
- Ex. 3. Two equal logarithmic spirals.
- Ex. 4. $\left\{ \begin{array}{l} \text{Circle whose equation is} \\ \text{Curve whose equation is} \end{array} \right. \quad \begin{array}{l} r = 2a \cos \theta. \\ \theta = \sqrt{2 \frac{a}{r} - 1} + \text{versin}^{-1} \frac{r}{a} \end{array}$
- Ex. 5. $\left\{ \begin{array}{l} \text{Cardioid whose equation is} \\ \text{Curve whose equation is} \end{array} \right. \quad \begin{array}{l} r = 2a(1 + \cos \theta). \\ \theta = \sin^{-1} \frac{r}{a} + \log \frac{r}{\sqrt{a^2 - r^2 + a}} \end{array}$
- Ex. 6. $\left\{ \begin{array}{l} \text{Conchoid,} \\ \text{Curve,} \end{array} \right. \quad \begin{array}{l} r = a(\sec \theta - 1). \\ \theta = \sqrt{1 - \frac{a^2}{r^2}} + \sec^{-1} \frac{r}{a} \end{array}$
- Ex. 7. $\left\{ \begin{array}{l} \text{Spiral of Archimedes,} \\ \text{Curve,} \end{array} \right. \quad \begin{array}{l} r = a\theta \\ \theta = \frac{r}{a} + \log \frac{r}{a} \end{array}$
- Ex. 8. $\left\{ \begin{array}{l} \text{Hyperbolic spiral,} \\ \text{Curve,} \end{array} \right. \quad \begin{array}{l} r = \frac{a}{\theta} \\ r = \frac{a}{e^{\theta} + 1} \end{array}$
- Ex. 9. $\left\{ \begin{array}{l} \text{Ellipse whose equation is,} \\ \text{Curve,} \end{array} \right. \quad \begin{array}{l} r = a \frac{1}{2 + \cos \theta} \\ r = a \left(1 + 2(2 - \theta^2) \right) \end{array}$
- Ex. 10. $\left\{ \begin{array}{l} \text{Involute of circle,} \\ \text{Curve,} \end{array} \right. \quad \begin{array}{l} \theta = \sqrt{\frac{r^2}{a^2} - 1} \sec^{-1} \frac{r}{a} \\ \theta = \sqrt{\frac{r^2}{a^2} \pm 2 \frac{r}{a}} \pm \log \left(\frac{r}{a} \pm 1 + \sqrt{\frac{r^2}{a^2} \pm 2 \frac{r}{a}} \right) \end{array}$

5th. Examples of curves rolling on themselves.

Ex. 1. When the curve which rolls on itself is a circle, equation

$$r = a \cos \theta$$

the traced curve is a cardioid, equation $r = a(1 + \cos \theta)$.

Ex. 2. When it is the curve whose equation is

$$r = 2^n a \left(\cos \frac{\theta}{r} \right)^n$$

the equation of the traced curve is

$$r = 2^{n+1} a \left(\cos \frac{\theta}{n+1} \right)^{n+1}$$

Ex. 3. When it is the involute of the circle, the traced curve is the spiral of Archimedes.

Ex. 4. When it is a parabola, the focus traces the directrix, and the vertex traces the cissoid.

Ex. 5. When it is the hyperbolic spiral, the traced curve is the tractory of the circle.

Ex. 6. When it is the polar catenary, the equation of the traced curve is

$$\vartheta = \sqrt{\frac{2a}{r} - 1} - \text{versin}^{-1} \frac{r}{a}$$

Ex. 7. When it is the curve whose equation is

$$\vartheta = \log \left(\sqrt{\frac{r^4}{a^4} - 1} + \frac{r^2}{a^2} \right) - \log \left(\sqrt{1 + \frac{a^4}{r^4} - \frac{a^2}{r^2}} \right)$$

the equation of the traced curve is, $r = a (e^{\vartheta} - e^{-\vartheta})$.

XXXVI.—*An Account of CARNOT's Theory of the Motive Power of Heat;** with Numerical Results deduced from REGNAULT's Experiments on Steam.† By WILLIAM THOMSON, Professor of Natural Philosophy in the University of Glasgow.

(Read January 2, 1849.)

1. The presence of heat may be recognised in every natural object; and there is scarcely an operation in nature which is not more or less affected by its all-pervading influence. An evolution and subsequent absorption of heat generally give rise to a variety of effects; among which may be enumerated, chemical combinations or decompositions; the fusion of solid substances; the vaporisation of solids or liquids; alterations in the dimensions of bodies, or in the statical pressure by which their dimensions may be modified; mechanical resistance overcome; electrical currents generated. In many of the actual phenomena of nature, several or all of these effects are produced together; and their complication will, if we attempt to trace the agency of heat in producing any individual effect, give rise to much perplexity. It will, therefore, be desirable, in laying the foundation of a physical theory of any of the effects of heat, to discover or to imagine phenomena free from all such complication, and depending on a definite thermal agency; in which the relation between the cause and effect, traced through the medium of certain simple operations, may be clearly appreciated. Thus it is that CARNOT, in accordance with the strictest principles of philosophy, enters upon the investigation of the theory of the motive power of heat.

2. The sole effect to be contemplated in investigating the motive power of heat is *resistance overcome*, or, as it is frequently called, "*work performed*," or "*mechanical effect*." The questions to be resolved by a complete theory of the subject are the following:

(1.) What is the precise nature of the thermal agency by means of which *mechanical effect* is to be produced, without effects of any other kind?

* Published in 1824, in a work entitled, "Réflexions sur la Puissance Motrice du Feu, et sur les Machines Propres à Développer cette Puissance. Par S. CARNOT." An account of CARNOT's Theory is also published in the *Journal d'Ecole Polytechnique*, vol. xiv., 1834, in a paper by MONS. CLAPEYRON.

† An account of the first part of a series of researches undertaken by MONS. REGNAULT, by order of the late French Government, for ascertaining the various physical data of importance in the theory of the steam-engine, has been recently published (under the title, "Relation des Expériences," &c.) in the *Mémoires de l'Institut*, of which it constitutes the twenty-first volume (1847). The second part of these researches has not yet been published.

(2.) How may the amount of this thermal agency necessary for performing a given quantity of work be estimated?

3. In the following paper I shall commence by giving a short abstract of the reasoning by which CARNOT is led to an answer to the first of these questions; I shall then explain the investigation by which, in accordance with his theory, the experimental elements necessary for answering the second question are indicated; and, in conclusion, I shall state the *data* supplied by REGNAULT'S recent observations on steam, and apply them to obtain, as approximately as the present state of experimental science enables us to do, a complete solution of the question.

1. On the nature of Thermal agency, considered as a motive power.

4. There are [at present known] two, and only two, distinct ways in which mechanical effect can be obtained from heat. One of these is by means of the alterations of volume which bodies may experience through the action of heat; the other is through the medium of electric agency. SEEBECK'S discovery of thermo-electric currents enables us at present to conceive of an electro-magnetic engine supplied from a thermal origin, being used as a motive power: but this discovery was not made until 1821, and the subject of thermo-electricity can only have been generally known in a few isolated facts, with reference to the electrical effects of heat upon certain crystals, at the time when CARNOT wrote. He makes no allusion to it, but confines himself to the method for rendering thermal agency available as a source of mechanical effect, by means of the expansions and contractions of bodies.

5. A body expanding or contracting under the action of force, may, in general, either produce mechanical effect by overcoming resistance, or receive mechanical effect by yielding to the action of force. The amount of mechanical effect thus developed will depend not only on the calorific agency concerned, but also on the alteration in the physical condition of the body. Hence, after allowing the volume and temperature of the body to change, we must restore it to its original temperature and volume; and then we may estimate the aggregate amount of mechanical effect developed as due solely to the thermal origin.

6. Now the ordinarily-received, and almost universally-acknowledged, principles with reference to "quantities of caloric" and "latent heat," lead us to conceive that, at the end of a cycle of operations, when a body is left in precisely its primitive physical condition, if it has absorbed any heat during one part of the operations, it must have given out again exactly the same amount during the remainder of the cycle. The truth of this principle is considered as axiomatic by CARNOT, who admits it as the foundation of his theory; and expresses himself in the following terms regarding it, in a note on one of the passages of his treatise.*

* CARNOT, p. 37.

"In our demonstrations we tacitly assume that after a body has experienced a certain number of transformations, if it be brought identically to its primitive physical state as to density, temperature, and molecular constitution, it must contain the same quantity of heat as that which it initially possessed; or, in other words, we suppose that the quantities of heat lost by the body under one set of operations are precisely compensated by those which are absorbed in the others. This fact has never been doubted; it has at first been admitted without reflection, and afterwards verified, in many cases, by calorimetrical experiments. To deny it would be to overturn the whole theory of heat, in which it is the fundamental principle. It must be admitted, however, that the chief foundations on which the theory of heat rests, would require a most attentive examination. Several experimental facts appear nearly inexplicable in the actual state of this theory."

7. Since the time when CARNOT thus expressed himself, the necessity of a most careful examination of the entire experimental basis of the theory of heat has become more and more urgent. Especially all those assumptions depending on the idea that heat is a *substance*, invariable in quantity; not convertible into any other element, and incapable of being *generated* by any physical agency; in fact the acknowledged principles of latent heat; would require to be tested by a most searching investigation before they ought to be admitted, as they usually have been, by almost every one who has been engaged on the subject, whether in combining the results of experimental research, or in general theoretical investigations.

8. The extremely important discoveries recently made by Mr JOULE of Manchester, that heat is evolved in every part of a closed electric conductor, moving in the neighbourhood of a magnet,* and that heat is *generated* by the friction of fluids in motion, seem to overturn the opinion commonly held that heat cannot be *generated*, but only produced from a source, where it has previously existed either in a sensible or in a latent condition.

* The evolution of heat in a fixed conductor, through which a galvanic current is sent from any source whatever, has long been known to the scientific world; but it was pointed out by Mr JOULE that we cannot infer from any previously-published experimental researches, the actual *generation* of heat when the current originates in electro-magnetic induction; since the question occurs, *is the heat which is evolved in one part of the closed conductor merely transferred from those parts which are subject to the inducing influence?* Mr JOULE, after a most careful experimental investigation with reference to this question, finds that it must be answered in the negative.—(See a paper "*On the Calorific Effects of Magneto-Electricity, and on the Mechanical Value of Heat*," by J. P. JOULE, Esq." Read before the British Association at Cork in 1843, and subsequently communicated by the Author to the *Philosophical Magazine*, vol. xxiii., pp. 263, 347, 435.)

Before we can finally conclude that heat is absolutely generated in such operations, it would be necessary to prove that the inducing magnet does not become lower in temperature, and thus compensate for the heat evolved in the conductor. I am not aware that any examination with reference to the truth of this conjecture has been instituted; but, in the case where the inducing body is a pure electro-magnet (without any iron), the experiments actually performed by Mr JOULE render the conclusion probable that the heat evolved in the wire of the electro-magnet is not affected by the inductive action, otherwise than through the reflected influence which increases the strength of its own current.

In the present state of science, however, no operation is known by which heat can be absorbed into a body without either elevating its temperature, or becoming latent, and producing some alteration in its physical condition; and the fundamental axiom adopted by CARNOT may be considered as still the most probable basis for an investigation of the motive power of heat; although this, and with it every other branch of the theory of heat may ultimately require to be reconstructed upon another foundation, when our experimental data are more complete. On this understanding, and to avoid a repetition of doubts, I shall refer to CARNOT'S fundamental principle, in all that follows, as if its truth were thoroughly established.

9. We are now led to the conclusion that the origin of motive power, developed by the alternate expansions and contractions of a body, must be found in the agency of heat entering the body and leaving it; since there cannot, at the end of a complete cycle, when the body is restored to its primitive physical condition, have been any absolute absorption of heat, and consequently no conversion of heat, or caloric, into mechanical effect; and it remains for us to trace the precise nature of the circumstances under which heat must enter the body, and afterwards leave it, so that mechanical effect may be produced. As an example, we may consider that machine for obtaining motive power from heat with which we are most familiar—the steam-engine.

10. Here, we observe, that heat enters the machine from the furnace, through the sides of the boiler, and that heat is continually abstracted by the water employed for keeping the condenser cool. According to CARNOT'S fundamental principle, the quantity of heat thus discharged, during a complete revolution (or double stroke) of the engine must be precisely equal to that which enters the water of the boiler;* provided the total mass of water and steam be invariable, and be restored to its primitive physical condition (which will be the case rigorously, if the condenser be kept cool by the external application of cold water, instead of by injection, as is more usual in practice), and if the condensed water be restored to the boiler at the end of each complete revolution. Thus, we perceive, that a certain quantity of heat is *let down* from a hot body, the metal of the boiler, to another body at a lower temperature, the metal of the condenser; and that there results from this transference of heat, a certain development of mechanical effect.

11. If we examine any other case in which mechanical effect is obtained from a thermal origin, by means of the alternate expansions and contractions of any substance whatever, instead of the water of a steam-engine, we find that a similar transference of heat is effected, and we may therefore answer the first question proposed, in the following manner:—

The thermal agency by which mechanical effect may be obtained, is the transference of heat from one body to another at a lower temperature.

* So generally is CARNOT'S principle tacitly admitted as an axiom, that its application in this case has never, so far as I am aware, been questioned by practical engineers.

II. On the measurement of Thermal Agency, considered with reference to its equivalent of mechanical effect.

12. A *perfect* thermo-dynamic engine of any kind, is a machine by means of which the greatest possible amount of mechanical effect can be obtained from a given thermal agency; and, therefore, if in any manner we can construct or imagine a perfect engine which may be applied for the transference of a given quantity of heat from a body at any given temperature, to another body, at a lower given temperature, and if we can evaluate the mechanical effect thus obtained, we shall be able to answer the question at present under consideration, and so to complete the theory of the motive power of heat. But whatever kind of engine we may consider with this view, it will be necessary for us to prove that it is a perfect engine; since the transference of the heat from one body to the other may be wholly, or partially, effected by conduction through a solid,* without the development of mechanical effect; and, consequently, engines may be constructed in which the whole, or any portion of the thermal agency is wasted. Hence it is of primary importance to discover the criterion of a perfect engine. This has been done by CARNOT, who proves the following proposition:—

13. *A perfect thermo-dynamic engine is such that, whatever amount of mechanical effect it can derive from a certain thermal agency; if an equal amount be spent in working it backwards, an equal reverse thermal effect will be produced.†*

14. This proposition will be made clearer by the applications of it which are given below (§ 29), in the cases of the air-engine and the steam-engine, than it could be by any general explanation; and it will also appear, from the nature of the operations described in those cases, and the principles of CARNOT's reasoning, that a perfect engine may be constructed with any substance of an indestructible texture as the alternately expanding and contracting medium. Thus we might conceive thermo-dynamic engines founded upon the expansions

* When "thermal agency" is thus spent in conducting heat through a solid, what becomes of the mechanical effect which it might produce? Nothing can be lost in the operations of nature—no energy can be destroyed. What effect then is produced in place of the mechanical effect which is lost? A perfect theory of heat imperatively demands an answer to this question; yet no answer can be given in the present state of science. A few years ago, a similar confession must have been made with reference to the mechanical effect lost in a fluid set in motion in the interior of a rigid closed vessel, and allowed to come to rest by its own internal friction; but in this case, the foundation of a solution of the difficulty has been actually found, in Mr JOULE's discovery of the generation of heat, by the internal friction of a fluid in motion. Encouraged by this example, we may hope that the very perplexing question in the theory of heat, by which we are at present arrested, will, before long, be cleared up.

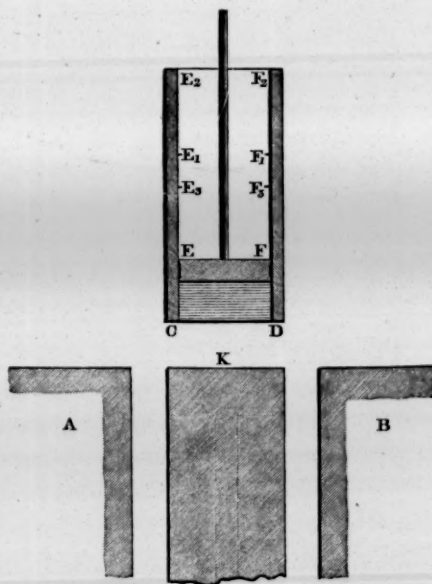
It might appear, that the difficulty would be entirely avoided, by abandoning CARNOT's fundamental axiom; a view which is strongly urged by Mr JOULE (at the conclusion of his paper "On the Changes of Temperature produced by the Rarefaction and Condensation of Air." *Phil. Mag.*, May 1845, vol. xxvi.) If we do so, however, we meet with innumerable other difficulties—insuperable without farther experimental investigation, and an entire reconstruction of the theory of heat, from its foundation. It is in reality to experiment that we must look—either for a verification of CARNOT's axiom, and an explanation of the difficulty we have been considering; or for an entirely new basis of the Theory of Heat.

† For a demonstration, see § 29, below.

and contractions of a perfectly elastic solid, or of a liquid ; or upon the alterations of volume experienced by substances, in passing from the liquid to the solid state,* each of which being perfect, would produce the same amount of mechanical effect from a given thermal agency ; but there are two cases which CARNOT has selected as most worthy of minute attention, because of their peculiar appropriateness for illustrating the general principles of his theory, no less than on account of their very great practical importance ; the steam-engine, in which the substance employed as the transferring medium is water, alternately in the liquid state, and in the state of vapour ; and the air-engine, in which the transference is effected by means of the alternate expansions and contractions of a medium, always in the gaseous state. The details of an actually practicable engine of either kind are not contemplated by CARNOT, in his general theoretical reasonings, but he confines himself to the ideal construction, in the simplest possible way in each case, of an engine in which the economy is perfect. He thus determines the degree of perfectibility which cannot be surpassed ; and, by describing a conceivable method of attaining to this perfection by an air-engine or a steam-engine, he points out the proper objects to be kept in view in the practical construction and working of such machines. I now proceed to give an outline of these investigations.

CARNOT'S Theory of the Steam-Engine.

15. Let CDF_2E_2 be a cylinder, of which the curved surface is perfectly impermeable to heat, with a piston also impermeable to heat, fitted in it ; while the fixed bottom CD , itself with no capacity for heat, is possessed of perfect conducting power. Let K be an impermeable stand, such that when the cylinder is placed upon it, the contents below the piston can neither gain nor lose heat. Let A and B be two bodies permanently retained at constant temperatures, S^0 and T^0 , respectively, of which the former is higher than the latter. Let the cylinder, placed on the impermeable stand, K , be partially filled with water, at the temperature S , of the body A , and (there being no air below it) let the piston be placed in a position $E F$, near the surface of the water. The



* A case minutely examined in another paper, to be laid before the Society at the present meeting.

pressure of the vapour above the water will tend to push up the piston, and must be resisted by a force applied to the piston,* till the commencement of the operations, which are conducted in the following manner.

(1.) The cylinder being placed on the body A, so that the water and vapour may be retained at the temperature S, *let the piston rise any convenient height $E E_1$, to a position $E_1 F_1$, performing work by the pressure of the vapour below it during its ascent.*

[During this operation a certain quantity, H, of heat, the amount of latent heat in the fresh vapour which is formed, is abstracted from the body A.]

(2.) The cylinder being removed, and placed on the impermeable stand K, *let the piston rise gradually, till, when it reaches a position $E_2 F_2$, the temperature of the water and vapour is T, the same as that of the body B.*

[During this operation the fresh vapour continually formed requires heat to become latent; and, therefore, as the contents of the cylinder are protected from any accession of heat, their temperature sinks.]

(3.) The cylinder being removed from K, and placed on B, *let the piston be pushed down, till, when it reaches the position $E_3 F_3$, the quantity of heat evolved and abstracted by B amounts to that which, during the first operation, was taken from A.*

[During this operation the temperature of the contents of the cylinder is retained constantly at T , and all the latent heat of the vapour which is condensed into water at the same temperature, is given out to B.]

(4.) The cylinder being removed from B, and placed on the impermeable stand, *let the piston be pushed down from $E_3 F_3$ to its original position EF.*

[During this operation, the impermeable stand preventing any loss of heat, the temperature of the water and air must rise continually, till (since the quantity of heat evolved during the third operation was precisely equal to that which was previously absorbed), at the conclusion it reaches its primitive value, S, in virtue of Carnot's fundamental axiom.]

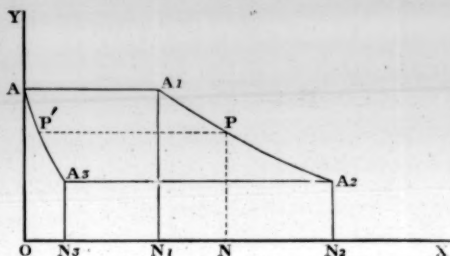
16. At the conclusion of this cycle of operations† the total thermal agency has been the *letting down* of H units of heat from the body A, at the temperature S, to B, at the lower temperature T; and the aggregate of the mechanical effect has been a certain amount of *work produced*, since during the ascent of the piston in the first and second operations, the temperature of the water and vapour, and therefore the pressure of the vapour on the piston, was on the whole higher than during the descent, in the third and fourth operations. It remains for us actually to evaluate this aggregate amount of work performed; and for this purpose the

* In all that follows, the pressure of the atmosphere on the upper side of the piston will be included in the applied forces, which, in the successive operations described, are sometimes overcome by the upward motion, and sometimes yielded to in the motion downwards. It will be unnecessary, in reckoning at the end of a cycle of operations, to take into account the work thus spent upon the atmosphere, and the restitution which has been made, since these precisely compensate for one another.

† In CARNOT'S work some perplexity is introduced with reference to the temperature of the water, which, in the operations he describes, is not brought back exactly to what it was at the commencement; but the difficulty which arises is explained by the author. No such difficulty occurs with reference to the cycle of operations described in the text, for which I am indebted to Mons. CLAPEYRON.

following graphical method of representing the mechanical effect developed in the several operations, taken from Mons. CLAPEYRON's paper, is extremely convenient.

17. Let $O X$ and $O Y$ be two lines at right angles to one another. Along $O X$ measure off distances $O N_1, N_1 N_2, N_2 N_3, N_3 O$, respectively proportional to the spaces described by the piston during the four successive operations described above; and, with reference to these four operations respectively, let the following constructions be made:—



(1.) Along $O Y$ measure a length $O A$, to represent the pressure of the saturated vapour at the temperature S ; and draw $A A_1$ parallel to $O X$, and let it meet an ordinate through N_1 in A_1 .

(2.) Draw a curve $A_1 P A$ such that, if $O N$ represent, at any instant during the second operation, the distance of the piston from its primitive position, $N P$ shall represent the pressure of the vapour at the same instant.

(3.) Through A_2 draw $A_2 A_3$ parallel to $O X$, and let it meet an ordinate through N_3 in A_3 .

(4.) Draw the curve $A_3 A$ such that the abscissa and ordinate of any point in it may represent respectively the distances of the piston from its primitive position, and the pressure of the vapour, at some instant during the fourth operation. The last point of this curve must, according to Carnot's fundamental principle, coincide with A , since the piston is, at the end of the cycle of operations, again in its primitive position, and the pressure of the vapour is the same as it was at the beginning.

18. Let us now suppose that the lengths, $O N_1, N_1 N_2, N_2 N_3$, and $N_3 O$, represent numerically the volumes of the spaces moved through by the piston during the successive operations. It follows that the mechanical effect obtained during the first operation will be numerically represented by the area $A A_1 N_1 O$; that is, the number of superficial units in this area will be equal to the number of "foot-pounds" of work performed by the ascending piston during the first operation. The work performed by the piston during the second operation will be similarly represented by the area $A_1 A_2 N_2 N_1$. Again, during the third operation a certain amount of work is spent on the piston, which will be represented by the area $A_2 A_3 N_3 N_2$; and lastly, during the fourth operation, work is spent in pushing the piston to an amount represented by the area $A_3 A O N_3$.

19. Hence the mechanical effect (represented by the area $O A A_1 A_2 N_2$) which was obtained during the first and second operations, exceeds the work (represented by $N_2 A_2 A_3 A O$) spent during the third and fourth, by an amount represented by the area of the quadrilateral figure $A A_1 A_2 A_3$; and, consequently, it

only remains for us to evaluate this area, that may determine the total mechanical effect gained in a complete cycle of operations. Now, from experimental data, at present nearly complete, as will be explained below, we may determine the length of the line $A_1 A_2$ for the given temperature S , and a given absorption H , of heat, during the first operation; and the length of $A_2 A_3$ for the given lower temperature T , and the evolution of the same quantity of heat during the fourth operation: and the curves $A_1 P A_2$, $A_3 P' A$ may be drawn as graphical representations of actual observations.* The figure being thus constructed, its area may be measured, and we are, therefore, in possession of a graphical method of determining the amount of mechanical effect to be obtained from any given thermal agency. As, however, it is merely the area of the figure which it is required to determine, it will not be necessary to be able to describe each of the curves $A_1 P A_2$, $A_3 P' A$, but it will be sufficient to know the difference of the abscissas corresponding to any equal ordinates in the two; and the following analytical method of completing the problem is the most convenient for leading to the actual numerical results.

20. Draw any line $P P'$ parallel to $O X$, meeting the curvilinear sides of the quadrilateral in P and P' . Let ξ denote the length of this line, and p its distance from $O X$. The area of the figure, according to the integral calculus, will be denoted by the expression

$$\int_{p_3}^{p_1} \xi \, dp,$$

where p_1 , and p_3 (the limits of integration indicated according to FOURIER'S notation) denote the lines $O A$, and $N_3 A_3$, which represent respectively the pressures during the first and third operations. Now, by referring to the construction described above, we see that ξ is the difference of the volumes below the piston at corresponding instants of the second and fourth operations, or instants at which the saturated steam and the water in the cylinder have the same pressure p , and, consequently, the same temperature which we may denote by t . Again, throughout the second operation the entire contents of the cylinder possess a greater amount of heat by H units than during the fourth; and, therefore, at any instant of the second operation there is as much more steam as contains H units of latent heat, than at the corresponding instant of the fourth operation. Hence, if k denote the latent heat in a unit of saturated steam at the temperature t , the volume of the steam at the two corresponding instants must differ by $\frac{H}{k}$. Now, if σ denote the ratio of the density of the steam to that of the water, the volume $\frac{H}{k}$ of steam will be formed from the volume $\sigma \frac{H}{k}$ of water; and, consequently, we have

* See Note at the end of this Paper.

for the difference of volumes of the entire contents at the corresponding instants,

$$\xi = (1 - \sigma) \frac{H}{k}.$$

Hence the expression for the area of the quadrilateral figure becomes

$$\int_{p_3}^{p_1} (1 - \sigma) \frac{H}{k} dp.$$

Now, σ , k , and p , being quantities which depend upon the temperature, may be considered as functions of t ; and it will be convenient to modify the integral so as to make t the independent variable. The limits will be from $t=T$ to $t=S$, and, if we denote by M the value of the integral, we have the expression

$$M = H \int_T^S (1 - \sigma) \frac{dp}{k} dt \dots \dots (1)$$

for the total amount of mechanical effect gained by the operations described above.

21. If the interval of temperatures be extremely small; so small that $\frac{dp}{k} \frac{dt}{dt}$ will not sensibly vary for values of t between T and S , the preceding expression becomes simply

$$M = (1 - \sigma) \frac{dp}{k} \cdot H (S - T) \dots \dots (2).$$

This might, of course, have been obtained at once, by supposing the breadth of the quadrilateral figure $A A_1 A_2 A$ to be extremely small compared with its length, and then taking for its area, as an approximate value, the product of the breadth into the line $A A_1$, or the line $A_3 A_2$, or any line of intermediate magnitude.

The expression (2) is rigorously correct for any interval $S - T$, if the mean value of $(1 - \sigma) \frac{dp}{k}$ for that interval be employed as the coefficient of $H (S - T)$.

CARNOT'S *Theory of the Air-Engine.*

22. In the ideal air-engine imagined by CARNOT four operations performed upon a mass of air or gas enclosed in a closed vessel of variable volume, constitute a complete cycle, at the end of which the medium is left in its primitive physical condition; the construction being the same as that which was described above for the steam-engine, a body A , permanently retained at the temperature S , and B at the temperature T ; an impermeable stand K ; and a cylinder and piston, which, in this case, contains a mass of air at the temperature S , instead of

water in the liquid state, at the beginning and end of a cycle of operations. The four successive operations are conducted in the following manner :—

(1.) The cylinder is laid on the body A, so that the air in it is kept at the temperature S; and the piston is allowed to rise, performing work.

(2.) The cylinder is placed on the impermeable stand K, so that its contents can neither gain nor lose heat, and the piston is allowed to rise farther, still performing work, till the temperature of the air sinks to T.

(3.) The cylinder is placed on B, so that the air is retained at the temperature T, and the piston is pushed down till the air gives out to the body B as much heat as it had taken in from A, during the first operation.

(4.) The cylinder is placed on K, so that no more heat can be taken in or given out, and the piston is pushed down to its primitive position.

23. *At the end of the fourth operation the temperature must have reached its primitive value S, in virtue of CARNOT'S axiom.*

24. Here, again, as in the former case, we observe that work is performed by the piston during the first two operations; and, during the third and fourth, work is spent upon it, but to a less amount, since the pressure is on the whole less during the third and fourth operations than during the first and second, on account of the temperature being lower. Thus, at the end of a complete cycle of operations, mechanical effect has been obtained; and the thermal agency from which it is drawn is the taking of a certain quantity of heat from A, and *letting it down*, through the medium of the engine, to the body B at a lower temperature.

25. To estimate the actual amount of effect thus obtained, it will be convenient to consider the alterations of volume of the mass of air in the several operations as extremely small. We may afterwards pass by the integral calculus, or, practically, by summation, to determine the mechanical effect whatever be the amplitudes of the different motions of the piston.

26. Let dq be the quantity of heat absorbed during the first operation, which is evolved again during the third; and let dv be the corresponding augmentation of volume which takes place while the temperature remains constant, as it does during the first operation.* The diminution of volume in the third operation must be also equal to dv , or only differ from it by an infinitely small

* Thus, $\frac{dq}{dv}$ will be the partial differential coefficient, with respect to v of that function of v and t , which expresses the quantity of heat that must be added to a mass of air when in a "standard" state (such as at the temperature zero, and under the atmospheric pressure), to bring it to the temperature t , and the volume v . That there is such a function, of two independent variables v and t , is merely an analytical expression of CARNOT'S fundamental axiom, as applied to a mass of air. The general principle may be analytically stated in the following terms :—If $M dv$ denote the accession of heat received by a mass of any kind, not possessing a destructible texture, when the volume is increased by dv , the temperature being kept constant, and if $N dt$ denote the amount of heat which must be supplied to raise the temperature by dt , without any alteration of volume; then $M dv + N dt$ must be the differential of a function of v and t .

quantity of the second order. During the second operation we may suppose the volume to be increased by an infinitely small quantity ϕ ; which will occasion a diminution of pressure, and a diminution of temperature, denoted respectively by ω and τ . During the fourth operation there will be a diminution of volume, and an increase of pressure and temperature, which can only differ, by infinitely small quantities of the second order, from the changes in the other direction, which took place in the second operation, and they also may, therefore, be denoted by ϕ , ω , and τ , respectively. The alteration of pressure, during the first and third operations, may at once be determined by means of MARIOTTE'S law, since, in them, the temperature remains constant. Thus, if, at the commencement of the cycle, the volume and pressure be v and p , they will have become $v + dv$ and $p \frac{v}{v + dv}$ at the end of the first operation. Hence the diminution of pressure, during the first operation, is $p - p \frac{v}{v + dv}$ or $p \frac{dv}{v + dv}$; and, therefore, if we neglect infinitely small quantities of the second order, we have $p \frac{dv}{v}$ for the diminution of pressure during the first operation; which, to the same degree of approximation, will be equal to the increase of pressure during the third. If $t + \tau$ and t be taken to denote the superior and inferior limits of temperature, we shall thus have for the volume, the temperature, and the pressure at the commencements of the four successive operations, and at the end of the cycle, the following values respectively:—

(1.)	$v,$	$t + \tau,$	$p;$
(2.)	$v + dv,$	$t + \tau,$	$p \left(1 - \frac{dv}{v}\right);$
(3.)	$v + dv + \phi,$	$t,$	$p \left(1 - \frac{dv}{v}\right) - \omega;$
(4.)	$v + \phi,$	$t,$	$p - \omega;$
(5.)	$v,$	$t + \tau,$	$p.$

Taking the mean of the pressures at the beginning and end of each operation, we find

(1.)	$p \left(1 - \frac{1}{2} \frac{dv}{v}\right)$
(2.)	$p \left(1 - \frac{dv}{v}\right) - \frac{1}{2} \omega$
(3.)	$p \left(1 - \frac{1}{2} \frac{dv}{v}\right) - \omega$
(4.)	$p - \frac{1}{2} \omega,$

which, as we are neglecting infinitely small quantities of the second order, will be

the expressions for the mean pressures during the four successive operations. Now, the mechanical effect gained or spent, during any of the operations, will be found by multiplying the mean pressure by the increase or diminution of volume which takes place; and we thus find

$$(1.) \quad p \left(1 - \frac{1}{2} \frac{dv}{v}\right) dv$$

$$(2.) \quad \left\{ p \left(1 - \frac{dv}{v}\right) - \frac{1}{2} \omega \right\} \varphi$$

$$(3.) \quad \left\{ p \left(1 - \frac{1}{2} \frac{dv}{v}\right) - \omega \right\} dv$$

$$(4.) \quad (p - \frac{1}{2} \omega) \varphi$$

for the amounts gained during the first and second, and spent during the third and fourth operations; and hence, by addition and subtraction, we find

$$\omega dv - p \varphi \frac{dv}{v}, \text{ or } (v \omega - p \varphi) \frac{dv}{v},$$

for the aggregate amount of mechanical effect gained during the cycle of operations. It only remains for us to express this result in terms of dq and τ , on which the given thermal agency depends. For this purpose, we remark that φ and ω are alterations of volume and pressure which take place along with a change of temperature τ , and hence, by the laws of compressibility and expansion, we may establish a relation* between them in the following manner.

Let p_0 be the pressure of the mass of air when reduced to the temperature zero, and confined in a volume v_0 ; then, whatever be v , the product $p_0 v_0$ will, by the law of compressibility, remain constant; and, if the temperature be elevated from 0 to $t + \tau$, and the gas be allowed to expand freely without any change of pressure, its volume will be increased in the ratio of 1 to $1 + E(t + \tau)$, where E is very nearly equal to .00366 (the centigrade scale of the air-thermometer being referred to), whatever be the gas employed, according to the researches of REGNAULT and of MAGNUS on the expansion of gases by heat. If, now, the volume be altered arbitrarily with the temperature continually at $t + \tau$, the product of the pressure and volume will remain constant; and, therefore, we have

$$p v = p_0 v_0 \{1 + E(t + \tau)\}.$$

Similarly

$$(p - \omega)(v + \varphi) = p_0 v_0 \{1 + E t\}.$$

Hence, by subtraction, we have

$$v \omega - p \varphi + \omega \varphi = p_0 v_0 E \tau,$$

or, neglecting the product $\omega \varphi$,

$$v \omega - p \varphi = p_0 v_0 E \tau.$$

* We might also investigate another relation, to express the fact that there is no accession or removal of heat during either the second or the fourth operation; but it will be seen that this will not affect the result in the text; although it would enable us to determine both φ and ω in terms of τ .

Hence, the preceding expression for mechanical effect, gained in the cycle of operations, becomes

$$p_0 v_0 \cdot E \tau \cdot \frac{dv}{v},$$

Or, as we may otherwise express it,

$$\frac{E p_0 v_0}{v \frac{dq}{dv}} \cdot dq \cdot \tau.$$

Hence, if we denote by M the mechanical effect due to H units of heat descending through the same interval τ , which might be obtained by repeating the cycle of operations described above, $\frac{H}{dq}$ times, we have

$$M = \frac{E p_0 v_0}{v \frac{dq}{dv}} \cdot H \tau \quad \dots \quad (3)$$

27. If the *amplitudes* of the operations had been finite, so as to give rise to an absorption of H units of heat during the first operation, and a lowering of temperature from S to T during the second, the amount of work obtained would have been found to be expressed by means of a double definite integral, thus;*

$$\left. \begin{aligned} M &= \int_0^H dq \int_T^S dt \cdot \frac{E p_0 v_0}{v \frac{dq}{dv}} \\ \text{or} \quad M &= E p_0 v_0 \int_0^H \int_T^S \frac{1}{v} \frac{dv}{dq} \cdot dt dq; \end{aligned} \right\} \dots \dots \dots (4),$$

this second form being sometimes more convenient.

28. The preceding investigations, being founded on the approximate laws of compressibility and expansion (known as the law of MARIOTTE and BOYLE, and the law of DALTON and GAY-LUSSAC), would require some slight modifications, to adapt them to cases in which the gaseous medium employed is such as to present sensible deviations from those laws. REGNAULT'S very accurate experiments shew that the deviations are insensible, or very nearly so, for the ordinary gases at ordinary pressures; although they may be considerable for a medium, such as

* This result might have been obtained by applying the usual notation of the integral calculus to express the area of the curvilinear quadrilateral, which, according to CLAPEYRON'S graphical construction, would be found to represent the entire mechanical effect gained in the cycle of operations of the air-engine. It is not necessary, however, to enter into the details of this investigation, as the formula (3), and the consequences derived from it, include the whole theory of the air-engine, in the best practical form; and the investigation of it which I have given in the text, will probably give as clear a view of the reasoning on which it is founded, as could be obtained by the graphical method, which, in this case, is not so valuable as it is from its simplicity in the case of the steam-engine.

sulphurous acid, or carbonic acid under high pressure, which approaches the physical condition of a vapour at saturation; and therefore, in general, and especially in practical applications to real air-engines, it will be unnecessary to make any modification in the expressions. In cases where it may be necessary, there is no difficulty in making the modifications, when the requisite data are supplied by experiment.

29.* Either the steam-engine or the air-engine, according to the arrangements described above, gives all the mechanical effect that can possibly be obtained from the thermal agency employed. For it is clear, that, in either case, the operations may be performed in the reverse order, with every thermal and mechanical effect reversed. Thus, in the steam-engine, we may commence by placing the cylinder on the impermeable stand, allow the piston to rise, performing work, to the position E, F_2 ; we may then place it on the body B, and allow it to rise, performing work, till it reaches E, F_1 ; after that the cylinder may be placed again on the impermeable stand, and the piston may be pushed down to E, F_1 ; and, lastly, the cylinder being removed to the body A, the piston may be pushed down to its primitive position. In this inverse cycle of operations, a certain amount of work has been spent, precisely equal, as we readily see, to the amount of mechanical effect gained in the direct cycle described above; and heat has been abstracted from B, and deposited in the body A, at a higher temperature, to an amount precisely equal to that which, in the direct cycle, was *let down* from A to B. Hence it is impossible to have an engine which will derive more mechanical effect from the same thermal agency, than is obtained by the arrangement described above; since, if there could be such an engine, it might be employed to perform, as a part of its whole work, the inverse cycle of operations, upon an engine of the kind we have considered, and thus to continually restore the heat from B to A, which has descended from A to B for working itself; so that we should have a complex engine, giving a residual amount of mechanical effect without any thermal agency, or alteration of materials, which is an impossibility in nature. The same reasoning is applicable to the air-engine; and we conclude, generally, that any two engines, constructed on the principles laid down above, whether steam-engines with different liquids, an air-engine and a steam-engine, or two air-engines with different gases, must derive the same amount of mechanical effect from the same thermal agency.

30. Hence, by comparing the amounts of mechanical effect obtained by the steam-engine and the air-engine from the letting down of the H units of heat from A at the temperature $(t + \tau)$ to B at t , according to the expressions (2) and (3), we have

* This paragraph is the demonstration referred to above, of the proposition stated in § 13; as it is readily seen that it is applicable to any conceivable kind of thermo-dynamic engine.

$$M = (1 - \sigma) \frac{dp}{dt} \cdot H \tau = \frac{E p_0 v_0}{v \frac{dq}{dv}} \cdot H \tau \quad \dots \quad (5).$$

If we denote the coefficient of $H \tau$ in these equal expressions by μ , which may be called "CARNOT'S coefficient," we have

$$\mu = (1 - \sigma) \frac{dp}{dt} = \frac{E p_0 v_0}{v \frac{dq}{dv}} \quad \dots \quad (6),$$

and we deduce the following very remarkable conclusions:—

(1.) For the saturated vapours of all different liquids, at the same temperature, the value of

$$(1 - \sigma) \frac{dp}{dt}$$

must be the same.

(2.) For any different gaseous masses, at the same temperature, the value of

$$\frac{E p_0 v_0}{v \frac{dq}{dv}}$$

must be the same.

(3.) The values of these expressions for saturated vapours and for gases, at the same temperature, must be the same.

31. No conclusion can be drawn *a priori* regarding the values of this coefficient μ for different temperatures, which can only be determined, or compared, by experiment. The results of a great variety of experiments, in different branches of physical science (Pneumatics and Acoustics), cited by CARNOT and by CLAPEYRON, indicate that the values of μ for low temperatures exceed the values for higher temperatures; a result amply verified by the continuous series of experiments performed by REGNAULT on the saturated vapour of water for all temperatures from 0° to 230° , which, as we shall see below, give values for μ gradually diminishing from the inferior limit to the superior limit of temperature. When, by observation, μ has been determined as a function of the temperature, the amount of mechanical effect, M , deducible from H units of heat descending from a body at the temperature S to a body at the temperature T , may be calculated from the expression,

$$M = H \int_T^S \mu dt \quad \dots \quad (7)$$

which is, in fact, what either of the equations (1) for the steam-engine, or (4) for the air-engine, becomes, when the notation μ , for CARNOT'S multiplier, is introduced.

The values of this integral may be practically obtained, in the most convenient manner, by first determining, from observation, the mean values of μ for the successive degrees of the thermometric scale, and then adding the values for all the degrees within the limits of the extreme temperatures S and T.*

32. The complete theoretical investigation of the motive power of heat is thus reduced to the experimental determination of the coefficient μ ; and may be considered as perfect, when, by any series of experimental researches whatever, we can find a value of μ for every temperature within practical limits. The special character of the experimental researches, whether with reference to gases, or with reference to vapours, necessary and sufficient for this object, is defined and restricted in the most precise manner, by the expressions (6) for μ , given above.

33. The object of REGNAULT'S great work, referred to in the title of this paper, is the experimental determination of the various physical elements of the steam-engine; and when it is complete, it will furnish all the *data* necessary for the calculation of μ . The valuable researches already published in a first part of that work, make known the latent heat of a given weight, and the pressure, of saturated steam for all temperatures between 0° and 230° cent. of the air-thermometer. Besides these data, however, the density of saturated vapour must be known, in order that k , the latent heat of a unit of volume, may be calculated from REGNAULT'S determination of the latent heat of a given weight.† Between the limits of 0° and 100°, it is probable, from various experiments which have been made, that the density of vapour follows very closely the simple laws which are so accurately verified by the ordinary gases;‡ and thus it may be calculated from REGNAULT'S table giving the pressure at any temperature within those limits. Nothing as yet is known with accuracy as to the density of saturated steam between 100° and 230°, and we must be contented at present to estimate it by calculation from REGNAULT'S table of pressures; although, when accurate experimental researches on the subject shall have been made, considerable deviations from the laws of BOYLE and DALTON, on which this calculation is founded, may be discovered.

* The results of these investigations are exhibited in Tables I. and II. below.

† It is, comparatively speaking, of little consequence to know accurately the value of σ , for the factor $(1-\sigma)$ of the expression for μ , since it is so small (being less than $\frac{1}{11,000}$ for all temperatures between 0° and 100°) that, unless all the data are known with more accuracy than we can count

upon at present, we might neglect it altogether, and take $\frac{dp}{dt}$ simply, as the expression for μ , without committing any error of important magnitude.

‡ This is well established, within the ordinary atmospheric limits, in REGNAULT'S *Études Météorologiques*, in the *Annales de Chimie*, vol. xv., 1846.

34. Such are the experimental data on which the mean values of μ for the successive degrees of the air-thermometer, from 0° to 230° , at present laid before the Royal Society, is founded. The unit of length adopted is the English foot; the unit of weight, the pound; the unit of work, a "foot-pound;" and the unit of heat that quantity which, when added to a pound of water at 0° , will produce an elevation of 1° in temperature. The mean value of μ for any degree is found to a sufficient degree of approximation, by taking, in place of σ , $\frac{dp}{dt}$, and k ; in the expression

$$(1-\sigma) \frac{\frac{dp}{dt}}{k};$$

the mean values of those elements; or, what is equivalent to the corresponding accuracy of approximation, by taking, in place of σ and k respectively, the mean of the values of those elements for the limits of temperature, and in place of $\frac{dp}{dt}$, the difference of the values of p , at the same limits.

35. In REGNAULT'S work (at the end of the eighth *Mémoire*), a table of the pressures of saturated steam for the successive temperatures 0° , 1° , 2° , . . . 230° , expressed in millimetres of mercury, is given. On account of the units adopted in this paper, these pressures must be estimated in pounds on the square foot, which we may do by multiplying each number of millimetres by 2.7896, the weight in pounds of a sheet of mercury, one millimetre thick, and a square foot in area.

36. The value of k , the latent heat of a cubic foot, for any temperature t , is found from λ , the latent heat of a pound of saturated steam, by the equation

$$k = \frac{p}{760} \cdot \frac{1 + .00366 \times 100}{1 + .00366 \times t} \times .036869 \cdot \lambda,$$

where p denotes the pressure in millimetres, and λ the latent heat of a pound of saturated steam; the values of λ being calculated by the empirical formula*

$$\lambda = (606.5 + 0.305 t) - (t + .00002 t^2 + 0.000000 t^3),$$

given by REGNAULT as representing, between the extreme limits of his observations, the latent heat of a unit weight of saturated steam.

* The part of this expression in the first vinculum (see REGNAULT, end of ninth *Mémoire*) is what is known as "the total heat" of a pound of steam, or the amount of heat necessary to convert a pound of water at 0° into a pound of saturated steam at t° ; which, according to "WATT'S law," thus approximately verified, would be constant. The second part, which would consist of the single term t , if the specific heat of water were constant for all temperatures, is the number of thermic units necessary to raise the temperature of a pound of water from 0° to t° , and expresses empirically the results of REGNAULT'S experiments on the specific heat of water (see end of the tenth *Mémoire*), described in the work already referred to.

Explanation of Table I.

37. The mean values of μ for the first, for the eleventh, for the twenty-first, and so on, up to the 231st* degree of the air-thermometer, have been calculated in the manner explained in the preceding paragraphs. These, and interpolated results, which must agree with what would have been obtained, by direct calculation from REGNAULT'S data, to three significant places of figures (and even for the temperatures between 0° and 100° , the experimental data do not justify us in relying on any of the results to a greater degree of accuracy), are exhibited in Table I.

To find the amount of mechanical effect due to a unit of heat, descending from a body at a temperature S to a body at T, if these numbers be integers, we have merely to add the values of μ in Table I. corresponding to the successive numbers.

$$T+1, T+2, \dots S-2, S-1,$$

Explanation of Table II.

38. The calculation of the mechanical effect, in any case, which might always be effected in the manner described in § 37 (with the proper modification for fractions of degrees, when necessary), is much simplified by the use of Table II., where the first number of Table I., the sum of the first and second, the sum of the first three, the sum of the first four, and so on, are successively exhibited. The sums thus tabulated are the values of the integrals

$$\int_0^1 \mu dt, \int_0^2 \mu dt, \int_0^3 \mu dt, \dots \int_0^{231} \mu dt;$$

and, if we denote $\int_0^t \mu dt$ by the letter M, Table II. may be regarded as a table of the values of M.

To find the amount of mechanical effect due to a unit of heat descending from a body at a temperature S to a body at T, if these numbers be integers, we have merely to subtract the value of M, for the number T + 1, from the value for the number S, given in Table II.

* In strictness, the 230th is the last degree for which the experimental data are complete; but the data for the 231st may readily be assumed in a sufficiently satisfactory manner.

TABLE I.* *Mean Values of μ for the successive Degrees of the Air-Thermometer from 0° to 230°.*

	μ		μ		μ		μ		μ
1°	4.960	48°	4.366	94°	3.889	140°	3.549	186°	3.309
2	4.946	49	4.355	95	3.880	141	3.543	187	3.304
3	4.932	50	4.343	96	3.871	142	3.537	188	3.300
4	4.918	51	4.331	97	3.863	143	3.531	189	3.295
5	4.905	52	4.319	98	3.854	144	3.525	190	3.291
6	4.892	53	4.308	99	3.845	145	3.519	191	3.287
7	4.878	54	4.296	100	3.837	146	3.513	192	3.282
8	4.865	55	4.285	101	3.829	147	3.507	193	3.278
9	4.852	56	4.273	102	3.820	148	3.501	194	3.274
10	4.839	57	4.262	103	3.812	149	3.495	195	3.269
11	4.826	58	4.250	104	3.804	150	3.490	196	3.265
12	4.812	59	4.239	105	3.796	151	3.484	197	3.261
13	4.799	60	4.227	106	3.788	152	3.479	198	3.257
14	4.786	61	4.216	107	3.780	153	3.473	199	3.253
15	4.773	62	4.205	108	3.772	154	3.468	200	3.249
16	4.760	63	4.194	109	3.764	155	3.462	201	3.245
17	4.747	64	4.183	110	3.757	156	3.457	202	3.241
18	4.735	65	4.172	111	3.749	157	3.451	203	3.237
19	4.722	66	4.161	112	3.741	158	3.446	204	3.233
20	4.709	67	4.150	113	3.734	159	3.440	205	3.229
21	4.697	68	4.140	114	3.726	160	3.435	206	3.225
22	4.684	69	4.129	115	3.719	161	3.430	207	3.221
23	4.672	70	4.119	116	3.712	162	3.424	208	3.217
24	4.659	71	4.109	117	3.704	163	3.419	209	3.213
25	4.646	72	4.098	118	3.697	164	3.414	210	3.210
26	4.634	73	4.088	119	3.689	165	3.409	211	3.206
27	4.621	74	4.078	120	3.682	166	3.404	212	3.202
28	4.609	75	4.067	121	3.675	167	3.399	213	3.198
29	4.596	76	4.057	122	3.668	168	3.394	214	3.195
30	4.584	77	4.047	123	3.661	169	3.389	215	3.191
31	4.572	78	4.037	124	3.654	170	3.384	216	3.188
32	4.559	79	4.028	125	3.647	171	3.380	217	3.184
33	4.547	80	4.018	126	3.640	172	3.375	218	3.180
34	4.535	81	4.009	127	3.633	173	3.370	219	3.177
35	4.522	82	3.999	128	3.627	174	3.365	220	3.173
36	4.510	83	3.990	129	3.620	175	3.361	221	3.169
37	4.498	84	3.980	130	3.614	176	3.356	222	3.165
38	4.486	85	3.971	131	3.607	177	3.351	223	3.162
39	4.474	86	3.961	132	3.601	178	3.346	224	3.158
40	4.462	87	3.952	133	3.594	179	3.342	225	3.155
41	4.450	88	3.943	134	3.586	180	3.337	226	3.151
42	4.438	89	3.934	135	3.579	181	3.332	227	3.148
43	4.426	90	3.925	136	3.573	182	3.328	228	3.144
44	4.414	91	3.916	137	3.567	183	3.323	229	3.141
45	4.402	92	3.907	138	3.561	184	3.318	230	3.137
46	4.390	93	3.898	139	3.555	185	3.314	231	3.134
47	4.378								

* The numbers here tabulated may also be regarded as, the actual values of μ for $t = \frac{1}{2}$, $t = 1\frac{1}{2}$, $t = 2\frac{1}{2}$, $t = 3\frac{1}{2}$, &c.

TABLE II. *Mechanical Effect in Foot-Pounds due to a Thermic Unit Centigrade, passing from a body, at any Temperature less than 230° to a body at 0°.*

Superior Limit of Temperature.	Mechanical Effect.	Superior Limit of Temperature.	Mechanical Effect.	Superior Limit of Temperature.	Mechanical Effect.	Superior Limit of Temperature.	Mechanical Effect.	Superior Limit of Temperature.	Mechanical Effect.
	Foot-pounds.		Foot-pounds.		Foot-pounds.		Foot-pounds.		Foot-pounds.
1°	4-960	48°	223-487	94°	412-545	140°	582-981	186°	740-310
2	9-906	49	227-842	95	416-425	141	586-524	187	743-614
3	14-838	50	232-185	96	420-296	142	590-061	188	746-914
4	19-756	51	236-516	97	424-159	143	593-592	189	750-209
5	24-661	52	240-835	98	428-013	144	597-117	190	753-500
6	29-553	53	245-143	99	431-858	145	600-636	191	756-787
7	34-431	54	249-439	100	435-695	146	604-099	192	760-069
8	39-296	55	253-724	101	439-524	147	607-656	193	763-347
9	44-148	56	257-997	102	443-344	148	611-157	194	766-621
10	48-987	57	262-259	103	447-156	149	614-652	195	769-890
11	53-813	58	266-509	104	450-960	150	618-142	196	773-155
12	58-625	59	270-748	105	454-756	151	621-626	197	776-416
13	63-424	60	274-975	106	458-544	152	625-105	198	779-673
14	68-210	61	279-191	107	462-324	153	628-578	199	782-926
15	72-983	62	283-396	108	466-096	154	632-046	200	786-175
16	77-743	63	287-590	109	469-860	155	635-508	201	789-420
17	82-490	64	291-773	110	473-617	156	638-965	202	792-661
18	87-225	65	295-945	111	477-366	157	642-416	203	795-898
19	91-947	66	300-106	112	481-107	158	645-862	204	799-131
20	96-656	67	304-256	113	484-841	159	649-302	205	802-360
21	101-353	68	308-396	114	488-567	160	652-737	206	805-585
22	106-037	69	312-525	115	492-286	161	656-167	207	808-806
23	110-709	70	316-644	116	495-998	162	659-591	208	812-023
24	115-368	71	320-752	117	499-702	163	663-010	209	815-236
25	120-014	72	324-851	118	503-399	164	666-424	210	818-446
26	124-648	73	328-939	119	507-088	165	669-833	211	821-652
27	129-269	74	333-017	120	510-770	166	673-237	212	824-854
28	133-878	75	337-084	121	514-445	167	676-636	213	828-052
29	138-474	76	341-141	122	518-113	168	680-030	214	831-247
30	143-058	77	345-188	123	521-174	169	683-419	215	834-438
31	147-630	78	349-225	124	525-428	170	686-803	216	837-626
32	152-189	79	353-253	125	529-075	171	690-183	217	840-810
33	156-736	80	357-271	126	532-715	172	693-558	218	843-990
34	161-271	81	361-280	127	536-348	173	696-928	219	847-167
35	165-793	82	365-279	128	539-975	174	700-293	220	850-340
36	170-303	83	369-269	129	543-595	175	703-654	221	853-509
37	174-801	84	373-249	130	547-209	176	707-010	222	856-674
38	179-287	85	377-220	131	550-816	177	710-361	223	859-836
39	183-761	86	381-181	132	554-417	178	713-707	224	862-994
40	188-223	87	385-133	133	558-051	179	717-049	225	866-149
41	192-673	88	389-076	134	561-597	180	720-386	226	869-300
42	197-111	89	393-010	135	565-176	181	723-718	227	872-448
43	201-537	90	396-935	136	568-749	182	727-046	228	875-592
44	205-951	91	400-851	137	572-316	183	730-369	229	878-733
45	210-353	92	404-758	138	575-877	184	733-687	230	881-870
46	214-743	93	408-656	139	579-432	185	737-001	231	885-004
47	219-121								

Note.—On the curves described in CLAPEYRON'S graphical method of exhibiting CARNOT'S Theory of the Steam-Engine.

39. At any instant when the temperature of the water and vapour is t , during the fourth operation (see above, § 16), the latent heat of the vapour must be precisely equal to the amount of heat that would be necessary to raise the temperature of the whole mass, if in the liquid state, from t to S .* Hence, if v' denote the volume of the vapour, c the mean capacity for heat of a pound of water between the temperatures S and t , and W the weight of the entire mass, in pounds, we have

$$k v' = c (S - t) W.$$

Again, the circumstances during the second operation are such that the mass of liquid and vapour possesses H units of heat more than during the fourth; and consequently, at the instant of the second operation, when the temperature is t , the volume v of the vapour will exceed v' by an amount of which the latent heat is H , so that we have

$$v = v' + \frac{H}{k}$$

40. Now, at any instant, the volume between the piston and its primitive position is less than the actual volume of vapour by the volume of the water evaporated. Hence, if x and x' denote the abscissæ of the curve at the instants of the second and fourth operations respectively, when the temperature is t , we have

$$x = v - \sigma v, \quad x' = v' - \sigma v',$$

and, therefore, by the preceding equations,

$$x = \frac{1 - \sigma}{k} \{H + c (S - t) W\} \quad \dots \quad (a)$$

$$x' = \frac{1 - \sigma}{k} c (S - t) W \quad \dots \quad (b)$$

These equations, along with

$$y = y' = p \quad \dots \quad (c)$$

enable us to calculate, from the data supplied by REGNAULT, the abscissa and ordinate for each of the curves described above (§ 17), corresponding to any as-

* For, at the end of the fourth operation, the whole mass is liquid, and at the temperature t . Now, this state might be arrived at by first compressing the vapour into water at the temperature t , and then raising the temperature of the liquid to S ; and however this state may be arrived at, there cannot, on the whole, be any heat added to or subtracted from the contents of the cylinder, since, during the fourth operation, there is neither gain nor loss of heat. This reasoning is, of course, founded on CARNOT'S fundamental principle, which is tacitly assumed in the commonly-received ideas connected with "WATT'S law," the "latent heat of steam," and "the total heat of steam."

sumed temperature t . After the explanations of §§ 33, 34, 35, 36, it is only necessary to add that c is a quantity of which the value is very nearly unity, and would be exactly so were the capacity of water for heat the same at every temperature as it is between 0° and 1° ; and that the value of $c(S-t)$, for any assigned values of S and t , is found, by subtracting the number corresponding to t from the number corresponding to s , in the column headed "*Nombre des unités de chaleur abandonnées par un kilogramme d'eau en descendant de T° à 0°* ", of the last table (at the end of the Tenth Mémoire) of REGNAULT'S work. By giving S the value 230° , and by substituting successively 220, 210, 200, &c., for t , values for x , y , x' , y' , have been found, which are exhibited in the following Table:—

Temperatures. t	Volumes to be described by the piston, to complete the fourth operation. x'	Volumes from the primitive position of the piston to those occupied at instants of the second operation. x	Pressures of saturated steam, in pounds on the square foot. $y = y' = p$
0°	1269.W	$x' + 5.409.H$	12.832
10	639.6.W	$x' + 2.847.H$	25.567
20	337.3.W	$x' + 1.571.H$	48.514
30	185.5.W	$x' + .9062.H$	88.007
40	105.9.W	$x' + .5442.H$	153.167
50	62.62.W	$x' + .3392.H$	256.595
60	38.19.W	$x' + .2188.H$	415.070
70	21.94.W	$x' + .1456.H$	650.240
80	15.38.W	$x' + .09962.H$	989.318
90	10.09.W	$x' + .06994.H$	1465.80
100	6.744.W	$x' + .05026.H$	2120.11
110	4.578.W	$x' + .03688.H$	2999.87
120	3.141.W	$x' + .02758.H$	4160.10
130	2.176.W	$x' + .02098.H$	5663.70
140	1.519.W	$x' + .01625.H$	7581.15
150	1.058.W	$x' + .01271.H$	9990.26
160	0.7369.W	$x' + .01010.H$	12976.2
170	0.5085.W	$x' + .008116.H$	16630.7
180	0.3454.W	$x' + .006592.H$	21051.5
190	0.2267.W	$x' + .005406.H$	26341.5
200	0.1409.W	$x' + .004472.H$	32607.7
210	0.0784.W	$x' + .003729.H$	39960.7
220	0.3310.W	$x' + .003130.H$	48512.4
230	0	$x' + .002643.H$	58376.6

Appendix.

(Read April 30, 1849.)

41. In p. 30, some conclusions drawn by CARNOT from his general reasoning were noticed; according to which it appears, that if the value of μ for any temperature is known, certain information may be derived with reference to the saturated vapour of any liquid whatever, and, with reference to any gaseous mass, without the necessity of experimenting upon the specific medium considered. Nothing in the whole range of Natural Philosophy is more remarkable than the establishment of general laws by such a process of reasoning. We have seen, however, that doubt may exist with reference to the truth of the axiom on which the entire theory is founded, and it therefore becomes more than a matter of mere curiosity to put the inferences deduced from it to the test of experience. The importance of doing so was clearly appreciated by CARNOT; and, with such data as he had from the researches of various experimenters, he tried his conclusions. Some very remarkable propositions which he derives from his Theory, coincide with DULONG and PETIT's subsequently-discovered experimental laws with reference to the heat developed by the compression of a gas; and the experimental verification is therefore in this case (so far as its accuracy could be depended upon) decisive. In other respects, the data from experiment were insufficient, although, so far as they were available as tests, they were confirmatory of the theory.

42. The recent researches of REGNAULT add immensely to the experimental data available for this object, by giving us the means of determining with considerable accuracy the values of μ within a very wide range of temperature, and so affording a trustworthy standard for the comparison of isolated results at different temperatures, derived from observations in various branches of physical science.

In the first section of this Appendix the Theory is tested, and shewn to be confirmed by the comparison of the values of μ found above, with those obtained by CARNOT and CLAPEYRON from the observations of various experimenters on air, and the vapours of different liquids. In the second and third sections some striking confirmations of the theory arising from observations by DULONG, on the specific heat of gases, and from Mr JOULE's experiments on the heat developed by the compression of air, are pointed out; and in con-

clusion, the actual methods of obtaining mechanical effect from heat are briefly examined with reference to their economy.

I. *On the values of μ derived by Carnot and Clapeyron from observations on Air, and on the Vapours of various liquids.*

43. In CARNOT'S work, p. 80-82, the mean value of μ between 0° and 1° is derived from the experiments of DELAROCHE and BERARD on the specific heat of gases, by a process approximately equivalent to the calculation of the value of $\frac{E p_0 v_0}{v \frac{dq}{dv}}$ for the temperature $\frac{1}{2}^\circ$. There are also, in the same work, determinations

of the values of μ from observations on the vapours of alcohol and water; but a table given in M. CLAPEYRON'S paper, of the values of μ derived from the data supplied by various experiments with reference to the vapours of ether, alcohol, water, and oil of turpentine, at the respective boiling-points of these liquids, afford us the means of comparison through a more extensive range of temperature. In the cases of alcohol and water, these results ought of course to agree with those of CARNOT. There are, however, slight discrepancies which must be owing to the uncertainty of the experimental data.* In the following table, CARNOT'S results with reference to air, and CLAPEYRON'S results with reference to the four different liquids, are exhibited, and compared with the values of μ which have been given above (Table I.) for the same temperatures, as derived from REGNAULT'S observations on the vapour of water.

Names of the Media.	Temperatures.	Values of μ	Values of μ deduced from Regnault's Observations.	Differences.
Air,	0.5°	(CARNOT) 4.377	4.960	.383
Sulphuric Ether,	(Boiling point) 35.5°	(CLAPEYRON) 4.478	4.510	.032
Alcohol,	78.8	3.963	4.030	.071
Water,	100	3.658	3.837	.179
Essence of Turpentine,	156.8	3.530	3.449	-.081

44. It may be observed that the discrepancies between the results founded on the experimental data supplied by the different observers with reference to water at the boiling-point, are greater than those which are presented between the results deduced from any of the other liquids, and water at the other temperatures; and we may therefore feel perfectly confident that the verification is com-

* Thus, from CARNOT'S calculations, we find, in the case of alcohol, 4.035; and in the case of water, 3.648, instead of 3.963, and 3.658, which are CLAPEYRON'S results in the same cases.

plete to the extent of accuracy of the observations.* The considerable discrepancy presented by CARNOT'S result, deduced from experiments on air, is not to be wondered at when we consider the very uncertain nature of his data.

45. The fact of the gradual decrease of μ through a very extensive range of temperature, being indicated both by REGNAULT'S continuous series of experiments, and by the very varied experiments on different media, and in different branches of Physical Science, must be considered as a striking verification of the theory.

11. On the Heat developed by the compression of Air.

46. Let a mass of air, occupying initially a given volume V , under a pressure P , at a temperature t , be compressed to a less volume V' , and allowed to part with heat until it sinks to its primitive temperature t . The quantity of heat which is evolved may be determined, according to CARNOT'S theory, when the particular value of μ , corresponding to the temperature t , is known. For, by equation § 30, equation (6), we have

$$v \frac{dq}{dv} = \frac{E p_0 v_0}{\mu},$$

where dq is the quantity of heat absorbed, when the volume is allowed to increase from v to $v + dv$; or the quantity evolved by the reverse operation. Hence we deduce

$$dq = \frac{E p_0 v_0}{\mu} \frac{dv}{v} \dots \dots (8),$$

Now, $\frac{E p_0 v_0}{\mu}$ is constant, since the temperature remains unchanged; and therefore, we may at once integrate the second number. By taking it between the limits V' and V , we thus find

$$Q = \frac{E p_0 v_0}{\mu} \log \frac{V}{V'} \dots \dots (9),$$

where Q denotes the required amount of heat, evolved by the compression from V to V' . This expression may be modified by employing the equations $PV = P' V' = p_0 v_0 (1 + E t)$; and we thus obtain

$$Q = \frac{EPV}{\mu(1 + Et)} \log \frac{V}{V'} = \frac{EP'V'}{\mu(1 + Et)} \log \frac{V}{V'} \dots \dots (10)$$

* A still closer agreement must be expected, when more accurate experimental data are afforded with reference to the other media. MONS. REGNAULT informs me that he is engaged in completing some researches, from which we may expect, possibly before the end of the present year, to be furnished with all the data for five or six different liquids which we possess at present for water. It is therefore to be hoped that, before long, a most important test of the validity of CARNOT'S theory will be afforded.

† The Napierian logarithm of $\frac{V}{V'}$ is here understood.

From this result we draw the following conclusion :—

47. *Equal volumes of all elastic fluids, when compressed to smaller equal volumes, disengage equal quantities of heat.*

This extremely remarkable theorem of CARNOT'S was independently laid down as a probable experimental law by DULONG, in his "*Recherches sur la Chaleur Spécifique des Fluides Élastiques*," and it therefore affords a most powerful confirmation of the theory.*

48. In some very remarkable researches made by Mr JOULE upon the heat developed by the compression of air, the quantity of heat produced in different experiments has been ascertained with reference to the amount of work spent in the operation. To compare the results which he has obtained with the indications of theory, let us determine the amount of work necessary actually to produce the compression considered above.

49. In the first place, to compress the gas from the volume $v + dv$ to v , the work required is $p dv$, or, since $p v = p_0 v_0 (1 + E t)$,

$$p_0 v_0 (1 + E t) \frac{dv}{v}.$$

Hence, if we denote by W the total amount of work necessary to produce the compression from V to V' , we obtain, by integration,

$$W = p_0 v_0 (1 + E t) \log \frac{V}{V'}.$$

Comparing this with the expression above, we find

$$\frac{W}{Q} = \frac{\mu(1 + E t)}{E} \dots \dots (11)$$

50. Hence we infer that

(1.) The amount of work necessary to produce a unit of heat by the compression of a gas, is the same for all gases at the same temperature.

(2.) And that the quantity of heat evolved in all circumstances, when the temperature of the gas is given, is proportional to the amount of work spent in the compression.

* CARNOT varies the statement of his theorem, and illustrates it in a passage, pp. 52, 53, of which the following is a translation :—

"When a gas varies in volume without any change of temperature, the quantities of heat absorbed or evolved by this gas are in arithmetical progression, if the augmentation or diminutions of volume are in geometrical progression.

"When we compress a litre of air maintained at the temperature 10° , and reduce it to half a litre, it disengages a certain quantity of heat. If, again, the volume be reduced from half a litre to a quarter of a litre, from a quarter to an eighth, and so on, the quantities of heat successively evolved will be the same.

"If, in place of compressing the air, we allow it to expand to two litres, four litres, eight litres, &c., it will be necessary to supply equal quantities of heat to maintain the temperature always at the same degree."

51. The expression for the amount of work necessary to produce a unit of heat is

$$\frac{\mu(1 + Et)}{E},$$

and therefore REGNAULT'S experiments on steam are available to enable us to calculate its value for any temperature. By finding the values of μ at 0° , 10° , 20° , &c., from Table I., and by substituting successively the values 0, 10, 20, &c., for t , the following results have been obtained.

Table of the Values of $\frac{\mu(1 + Et)}{E}$,

Work requisite to produce a unit of Heat by the com- pression of a Gas.	Temperature of the Gas.	Work requisite to produce a unit of Heat by the com- pression of a Gas.	Temperature of the Gas.
Ft.-lbs.	$^\circ$	Ft.-lbs.	$^\circ$
1357.1	0	1446.4	120
1368.7	10	1455.8	130
1379.0	20	1465.3	140
1388.0	30	1475.8	150
1395.7	40	1489.2	160
1401.8	50	1499.0	170
1406.7	60	1511.3	180
1412.0	70	1523.5	190
1417.6	80	1536.5	200
1424.0	90	1550.2	210
1430.6	100	1564.0	220
1438.2	110	1577.8	230

Mr JOULE'S experiments were all conducted at temperatures from 50° to about 60° Fahr., or from 10° to 16° cent.; and, consequently, although some irregular differences in the results, attributable to errors of observation inseparable from experiments of such a very difficult nature are presented, no regular dependence on the temperature is observable. From three separate series of experiments, Mr JOULE deduces the following numbers for the work, in foot-pounds, necessary to produce a thermic unit Fahrenheit by the compression of a gas.

820, 814, 760.

Multiplying these by 1.8, to get the corresponding number for a thermic unit centigrade, we find

1476, 1465, and 1368.

The largest of these numbers is most nearly conformable with Mr JOULE'S views of the relation between such experimental "equivalents," and others which he obtained in his electro-magnetic researches; but the smallest agrees almost perfectly with the indications of CARNOT'S theory; from which, as exhibited in the preceding Table, we should expect, from the temperature in Mr JOULE'S experiments, to find a number between 1369 and 1379 as the result.

III. On the Specific-Heats of Gases.

52. The following proposition is proved by CARNOT as a deduction from his general theorem regarding the specific heats of gases.

The excess of the specific heat under a constant pressure above the specific heat at a constant volume, is the same for all gases at the same temperature and pressure.*

53. To prove this proposition, and to determine an expression for the "excess" mentioned in its enunciation, let us suppose a unit of volume of a gas to be elevated in temperature by a small amount, τ . The quantity of heat required to do this will be $A\tau$, if A denote the specific heat at a constant volume. Let us next allow the gas to expand without going down in temperature, until its pressure becomes reduced to its primitive value. The expansion which will take place will be $\frac{E\tau}{1+E\tau}$, if the temperature be denoted by t ; and hence, by (8), the quantity of heat that must be supplied, to prevent any lowering of temperature, will be

$$\frac{E p_0 v_0}{\mu} \cdot \frac{E\tau}{1+E\tau}, \quad \text{or} \quad \frac{E^2 p}{\mu(1+E\tau)^2} \tau.$$

Hence, the total quantity added is equal to

$$A\tau + \frac{E^2 p}{\mu(1+E\tau)^2} \tau$$

But, since B denotes the specific heat under constant pressure, the quantity of heat requisite to bring the gas into this state, from its primitive condition, is equal to $B\tau$; and hence we have

$$B = A + \frac{E^2 p}{\mu(1+E\tau)^2} \dots \dots \dots (12)$$

IV. Comparison of the Relative advantages of the Air-Engine and Steam-Engine.

54. In the use of water-wheels for motive power, the economy of the engine depends not only upon the excellence of its adaptation for actually transmitting any given quantity of water through it, and producing the equivalent of work, but upon turning to account the entire available fall; so, as we are taught by CARNOT, the object of a thermodynamic engine is to economize in the best possible way the transference of all the heat evolved, from bodies at the temperature of the source, to bodies at the lowest temperature at which the heat can be discharged. With reference then to any engine of the kind, there will be two points to be considered.

(1.) The extent of the *fall* utilised.

(2.) The economy of the engine, with the fall which it actually uses.

55. In the first respect, the air-engine, as CARNOT himself points out, has a

* Or the capacity of a unit of volume for heat.

vast advantage over the steam-engine; since the temperature of the hot part of the machine may be made very much higher in the air-engine than would be possible in the steam-engine, on account of the very high pressure produced in the boiler, by elevating the temperature of the water which it contains to any considerable extent above the atmospheric boiling point. On this account, a "perfect air-engine" would be a much more valuable instrument than a "perfect steam-engine."*

Neither steam-engines nor air-engines, however, are nearly perfect; and we do not know in which of the two kinds of machine the nearest approach to perfection may be actually attained. The beautiful engine invented by Mr STIRLING of Galston, may be considered as an excellent beginning for the air-engine;† and it is only necessary to compare this with NEWCOMEN'S steam-engine, and consider what WATT has effected, to give rise to the most sanguine anticipations of improvement.

V. *On the Economy of actual Steam-Engines.*

56. The steam-engine being universally employed at present as the means for deriving motive power from heat, it is extremely interesting to examine, according to CARNOT'S theory, the economy actually attained in its use. In the first place, we remark that, out of the entire "fall" from the temperature of the coals to that of the atmosphere, it is only part—that from the temperature of the boiler to the temperature of the condenser—that is made available; while the very great fall from the temperature of the burning coals to that of the boiler, and the comparatively small fall from the temperature of the condenser to that of the atmosphere, are entirely lost as far as regards the mechanical effect which it is desired to obtain. We infer from this, that the temperature of the boiler ought to be kept as high as, according to the strength, is consistent with safety, while that of the condenser ought to be kept as nearly down at the atmospheric temperature as possible. To take the entire benefit of the actual fall, CARNOT shewed that the "principle of expansion" must be pushed to the utmost.‡

* CARNOT suggests a combination of the two principles, with air as the medium for receiving the heat at a very high temperature from the furnace; and a second medium, alternately in the state of saturated vapour and liquid water, to receive the heat, discharged at an intermediate temperature from the air, and transmit it to the coldest part of the apparatus. It is possible that a complex arrangement of this kind might be invented, which would enable us to take the heat at a higher temperature, and discharge it at a lower temperature than would be practicable in any simple air-engine or simple steam-engine. If so, it would no doubt be equally possible, and perhaps more convenient, to employ steam alone, but to use it at a very high temperature not in contact with water in the hottest part of the apparatus, instead of, as in the steam-engine, always in a saturated state.

† It is probably this invention to which CARNOT alludes in the following passage (p. 112):—"Il a été fait, dit-on, tout récemment en Angleterre des essais heureux sur le développement de la puissance motrice par l'action de la chaleur sur l'air atmosphérique. Nous ignorons entièrement ne quoi ces essais ont consisté, si toutefois ils sont réels."

‡ From this point of view, we see very clearly how imperfect is the steam-engine, even after all WATT'S improvements. For to "push the principle of expansion to the utmost," we must allow the

57. To obtain some notion of the economy which has actually been obtained, we may take the alleged performances of the best Cornish engines, and some other interesting practical cases as examples.*

(1.) The engine of the *Fowey Consols mine* was reported, in 1845, to have given 125,089,000 foot-pounds of effect, for the consumption of one bushel or 94 lbs. of coals. Now, the average amount evaporated from Cornish boilers, by one pound of coal, is $8\frac{1}{2}$ lbs. of steam; and hence, for each pound of steam evaporated 156.556 foot-pounds of work are produced.

The pressure of the saturated steam in the boiler may be taken as $3\frac{1}{2}$ atmospheres;† and, consequently, the temperature of the water will be 140° . Now (REGNAULT, end of Memoire X.), the latent heat of a pound of saturated steam at 140° is 508, and since, to compensate for each pound of steam removed from the boiler in the working of the engine, a pound of water, at the temperature of the condenser, which may be estimated at 30° , is introduced from the hot well; it follows that 618 units of heat are introduced to the boiler for each pound of water evaporated. But the work produced, for each pound of water evaporated, was found above to be 156.556 foot-pounds. Hence, $\frac{156.556}{618}$, or 253 foot-pounds is the amount of work produced for each unit of heat transmitted through the Fowey Consols engine. Now, in Table II., we find 583.0 as the theoretical effect due to a unit descending from 140° to 0° , and 143 as the effect due to a unit descending from 30° to 0° . The difference of these numbers, or 440,‡ is the number of foot-pounds of work that a *perfect* engine with its boiler at 140° , and its condenser at 30° would produce for each unit of heat transmitted. Hence, the Fowey Consols engine, during the experiments reported on, performed $\frac{253}{440}$ of its theoretical duty, or $57\frac{1}{2}$ per cent.

(2.) The best duty on record, as performed by an engine at work (not for merely experimental purposes), is that of TAYLOR'S engine, at the United mines, which, in 1840, worked regularly, for several months, at the rate of 98,000,000 foot-pounds for each bushel of coals burned. This is $\frac{98}{125}$, or .784 of the experimental

steam, before leaving the cylinder, to expand until its pressure is the same as that of the vapour in the condenser. According to "WATT'S law," its temperature would then be the same as (actually a little above, as REGNAULT has shewn) that of the condenser, and hence the steam-engine worked in this most advantageous way, has in reality the very fault that WATT found in NEWCOMEN'S engine. This defect is partially remedied by HORNBLOWER'S system of using a separate expansion cylinder, an arrangement, the advantages of which did not escape CARNOT'S notice, although they have not been recognised extensively among practical engineers, until within the last few years.

* I am indebted to the kindness of Professor GORDON of Glasgow, for the information regarding the various cases given in the text.

† In different Cornish engines, the pressure in the boiler is from $2\frac{1}{2}$ to 5 atmospheres; and, therefore, as we find from REGNAULT'S table of the pressure of saturated steam, the temperature of the water in the boiler must, in all of them, lie between 128° and 152° . For the better class of engines, the average temperature of the water in the boiler may be estimated at 140° , the corresponding pressure of steam being $3\frac{1}{2}$ atmospheres.

‡ This number agrees very closely with the number corresponding to the fall from 100° to 0° , given in Table II. Hence, the fall from 140° to 30° of the scale of the air-thermometer is equivalent, with reference to motive power, to the fall from 100° to 0° .

duty reported in the case of the Fowey Consols engine. Hence, the best useful work on record, is at the rate of 198·3 foot-pounds for each unit of heat transmitted, and is $\frac{198\cdot3}{440}$, or 45 per cent. of the theoretical duty, on the supposition that the boiler is at 140°, and the condenser at 30°.

(3.) French engineers contract (in Lille, in 1847, for example) to make engines for mill power which will produce 30,000 metre-lbs., or 98,427 foot-lbs. of work for each pound of steam used. If we divide this by 618, we find 159 foot-pounds for the work produced by each unit of heat. This is 36·1 per cent. of 440, the theoretical duty.*

(4.) English engineers have contracted to make engines and boilers which will require only $3\frac{1}{2}$ lbs. of the best coal per horse-power per hour. Hence, in such engines, each pound of coal ought to produce 565,700 foot-pounds of work, and if 7 lbs. of water be evaporated by each pound of coal, there would result 80,814 foot-pounds of work for each pound of water evaporated. If the pressure in the boiler be $3\frac{1}{2}$ atmospheres (temperature 140°) the amount of work for each unit of heat will be found, by dividing this by 618, to be 130·7 foot-pounds, which is $\frac{130\cdot7}{440}$ or 29·7 per cent. of the theoretical duty.†

(5.) The actual average of work performed by good Cornish engines and boilers is 55,000,000 foot-pounds for each bushel of coal, or less than half the experimental performance of the Fowey Consols engine, more than half the actual duty performed by the United Mines engine in 1840; in fact about 25 per cent. of the theoretical duty.

(6.) The average performances of a number of Lancashire engines and boilers have been recently found to be such as to require 12 lbs. of Lancashire coal per horse-power per hour (*i.e.*, for performing $60 \times 33,000$ foot-pounds) and of a number of Glasgow engines, such as to require 15 lbs. (of a somewhat inferior coal) for the same effect. There are, however, more than twenty large engines in Glasgow at present,‡ which work with a consumption of only $6\frac{1}{2}$ lbs. of dross, equivalent to 5 lbs. of the best Scotch, or 4 lbs. of the best Welsh coal, per horse-power

* It being assumed that the temperatures of the boiler and condenser are the same as those of the Cornish engines. If, however, the pressure be lower, two atmospheres, for instance, the numbers would stand thus: The temperature in the boiler would be only 121. Consequently, for each pound of steam evaporated, only 614 units of heat would be required; and, therefore, the work performed for each unit of heat transmitted would be 160·3 foot-pounds, which is *more* than according to the estimate in the text. On the other hand, the range of temperatures, or the fall utilised, is only from 131 to 30, instead of from 140 to 30°, and, consequently (Table II.), the theoretical duty for each unit of heat is only 371 foot-pounds. Hence, if the engine, to work according to the specification, requires a pressure of only 15 lbs. on the square inch (*i.e.*, a total steam pressure of two atmospheres), its performance is $\frac{160\cdot3}{371}$, or 43·2 per cent. of its theoretical duty.

† If, in this case again, the pressure required in the boiler to make the engine work according to the contract were only 15 lbs. on the square inch, we should have a different estimate of the economy, for which, see Table B, at the end of this paper.

‡ These engines are provided with separate expansive cylinders, which have been recently added to them by Mr M'NAUGHT of Glasgow.

per hour. The economy may be estimated from these data, as in the other cases, on the assumption which, with reference to these, is the most probable we can make, that the evaporation produced by a pound of best coal is 7 lbs. of steam.

58. The following Tables afford a synoptic view of the performances and theoretical duties in the various cases discussed above.

In Table A the numbers in the second column are found by dividing the numbers in the first by $8\frac{1}{2}$ in cases (1.), (2.), and (5.), and by 7 in cases (4.), (6.), and (7.), the estimated numbers of pounds of steam actually produced in the different boilers by the burning of 1 lb. of coal.

The numbers in the third column are found from those in the second, by dividing by 618, in Table A, and 614 in Table B, which are respectively the quantities of heat required to convert a pound of water taken from the hot well at 30° , into saturated steam, in the boiler, at 140° or at 121° .

With reference to the cases (3.), (4.), (6.), (7.), the hypothesis of Table B is probably in general nearer the truth than that of Table A. In (4.), (6.), and (7.), especially upon hypothesis B, there is much uncertainty as to the amount of evaporation that will be actually produced by 1 lb. of fuel. The assumption on which the numbers in the second column in Table B are calculated, is, that each pound of coal will send the same number of units of heat into the boiler whether hypothesis A or hypothesis B be followed. Hence, except in the case of the French contract, in which the *evaporation*, not the fuel, is specified, the numbers in the third column are the same as those in the third column of Table A.

TABLE A. *Various Engines in which the temperature of the Boiler is 140° , and that of the Condenser 30° .*

Theoretical Duty for each Unit of Heat transmitted, 440 foot-pounds.

CASES.	Work produced for each pound of coal consumed.	Work produced for each pound of water evaporated.	Work produced for each unit of heat transmitted.	Per cent- age of theoretical duty.
(1.) Fowey Consols Experiment, reported in 1845,	Foot-Pounds. 1,330,734	Foot-Pounds. 156,556	Foot-Pounds. 253	57.5
(2.) Taylor's Engine at the United Mines, working in 1840, . . . }	1,042,553	122,653	198.4	45.1
(3.) French Engines, according to contract, . . . }	* * * *	98,427	159	36.1
(4.) English Engines, according to contract, . . . }	565,700	80,814	130.8	29.7
(5.) Average actual performance of Cornish Engines, . . . }	585,106	68,836	111.3	25.3
(6.) Common Engines, consuming 12 lbs. of best coal per hour per horse-power, . . . }	165,000	23,571	38.1	9.6
(7.) Improved Engines with Expansion Cylinders, consuming an equivalent to 4 lbs. of best coal per horse-power per hour, . . . }	495,000	70,710	114.4	26

TABLE B. *Various Engines in which the Temperature of the Boilers is 121,* and that of the Condenser 30°.**Theoretical Duty for each Unit of Heat transmitted, 371 foot-pounds.*

CASES.	Work produced for each pound of coal con- sumed.	Work produced for each pound of water eva- porated.	Work produced for each unit of heat trans- mitted.	Per cent- age of theoreti- cal duty.
	Foot-Pounds.	Foot-Pounds.	Foot-Pounds.	
(3.) French Engines, according to contract, . . .	* * *	98,427	160·3	43·2
(4.) English Engines, according to contract, . . .	565,700	$\frac{3}{4} \times 80,814$	130·8	35
(6.) Common Engines, consuming 12 lbs. of coal per horse-power per hour, . . .	165,000	$\frac{3}{4} \times 23,571$	38·1	10·3
(7.) Improved Engines with expansion cylinders, consuming an equivalent to 4 lbs. best coal per horse-power per hour, . . .	495,000	$\frac{3}{4} \times 70,710$	114·4	30·7

* Pressure 15 lbs. on the square inch.

XXXVII.—*Theoretical Considerations on the Effect of Pressure in Lowering the Freezing Point of Water.* By JAMES THOMSON, Esq., of Glasgow. Communicated by Professor WILLIAM THOMSON.

(Read 2d January 1849.)

Some time ago my brother, Professor WILLIAM THOMSON, pointed out to me a curious conclusion to which he had been led, by reasoning on principles similar to those developed by CARNOT, with reference to the motive power of heat. It was, that *water at the freezing point may be converted into ice by a process solely mechanical, and yet without the final expenditure of any mechanical work.* This at first appeared to me to involve an impossibility, because water expands while freezing; and, therefore, it seemed to follow, that if a quantity of it were merely enclosed in a vessel with a moveable piston, and frozen, the motion of the piston, consequent on the expansion, being resisted by pressure, mechanical work would be given out without any corresponding expenditure; or, in other words, a perpetual source of mechanical work, commonly called a perpetual motion, would be possible. After farther consideration, however, the former conclusion appeared to be incontrovertible; but then, to avoid the absurdity of supposing that mechanical work could be got out of nothing, it occurred to me that it is necessary farther to conclude, that *the freezing point becomes lower as the pressure to which the water is subjected is increased.*

The following is the reasoning by which these conclusions are proved. Let there be supposed to be a cylinder, and a piston fitting water-tight to it, and capable of moving without friction. Let these be supposed to be formed of a substance which is a perfect non-conductor of heat; also, let the bottom of the cylinder be closed by a plate, supposed to be a perfect conductor, and to possess no capacity for heat. Now, to convert a given mass of ice into water without the expenditure of mechanical work, let this imaginary vessel be partly filled with air at 0° C., and let the end of it be placed in contact with an indefinite mass of water, a lake for instance, at the same temperature. Now, let the piston be pushed towards the bottom of the cylinder by pressure from some external reservoir of mechanical work, which, for the sake of fixing our ideas, may be supposed to be the hand of an operator. During this process the air in the cylinder would tend to become heated on account of the compression, but it is constrained to remain at 0° by being in communication with the lake at that temperature. The change, then, which takes place is, that a certain amount of work is given from the hand to the air, and a certain amount of heat is given from the air to the water of the lake. In the next place, let the bottom of the cylinder be placed in

contact with the mass of water at 0° , which is proposed to be converted into ice, and let the piston be allowed to move back to the position it had at the commencement of the first process. During this second process, the temperature of the air would tend to sink on account of the expansion, but it is constrained to remain constant at 0° by the air being in communication with the freezing water, which cannot change its temperature so long as any of it remains unfrozen. Hence, so far as the air and the hand are concerned, this process has been exactly the converse of the former one. Thus the air has expanded through the same distance through which it was formerly compressed; and, since it has been constantly at the same temperature during both processes, the law of the variation of its pressure with its volume must have been the same in both. From this it follows, that the hand has received back exactly the same amount of mechanical work in the second process as it gave out in the first. By an analogous reason it is easily shewn, that the air also has received again exactly the same amount of heat as it gave out during its compression; and, hence, it is now left in a condition the same as that in which it was at the commencement of the first process. *The only change which has been produced, then, is, that a certain quantity of heat has been abstracted from a small mass of water at 0° , and dispersed through an indefinite mass at the same temperature, the small mass having thus been converted into ice.* This conclusion, it may be remarked, might be deduced at once by the application, to the freezing of water, of the general principle developed by CARNOT, that no work is given out when heat passes from one body to another without a fall of temperature; or rather by the application of the converse of this, which of course equally holds good, namely, that no work requires to be expended to make heat pass from one body to another at the same temperature.

Next, to prove that the freezing point of water is lowered by an increase of the pressure to which the water is subjected:—Let a cylinder, of the same imaginary construction as that used in the foregoing demonstration, contain some air at 0° C. Let the bottom of the cylinder be placed in contact with the water of an indefinitely large lake, of which the temperature is above 0° by an infinitely small quantity; and let the air be subjected to compression by pressure applied by the hand to the piston. A certain amount of work is thus given from the hand to the air, and a certain amount of heat is given out from the air to the lake. Next, let the bottom of the cylinder be placed in communication with a small quantity of water at 0° , enclosed in a second imaginary cylinder similar in character to the first; and let this water be, at the commencement, subject merely to the atmospheric pressure. Let, however, resistance be offered by the hand to any motion of the piston of this second cylinder which may take place. Things being in this state, let the piston of the cylinder containing the air move back to its original position. During this process part of the heat of the air becomes latent on account of the increase of volume. Thus the temperature of the air, from being

above 0° , by an infinitely small quantity, instantly becomes absolutely 0° ; and afterwards, as the motion of the piston continues, the air absorbs heat from the mass of water in the second cylinder, part of the mass passing at the same time into the state of ice. Hence the whole mass expands; and therefore, on account of the resistance offered by the hand to the motion of the piston of the cylinder containing the mass, the internal pressure is increased, and a quantity of work, not infinitely small, is given out by the piston, and is received by the hand. Towards the end of this process, let the resistance offered by the hand gradually decrease till, just at the end (that is, when the piston of the air-cylinder has resumed its first position) it becomes nothing, and the pressure within the water-cylinder thus becomes again equal to that of the atmosphere. The temperature of the mass of partly frozen water must now be 0° , and the air in the other cylinder being in communication with this, must have the same temperature. The air is therefore, infinitely nearly at its original temperature, and it has its original volume. Hence it is now left in a state infinitely nearly the same as that in which it was at first. Farther, let the ice, which has been formed by the freezing of the water, be placed in contact with the lake till it melts, which it will really do since the lake is warmer than 0° , though only by an infinitely small quantity. Thus the mass of water is left in its original state, and it has been already shewn that the air is left infinitely nearly in its original state. Hence no work, except an infinitely small quantity, can have been absorbed or developed by any change on the air and water, which have been used. But a quantity of work not infinitely small has been given out by the piston of the water-cylinder to the hand; and therefore an equal quantity* of work must have been given from the hand to the air-piston, as there is no other way in which the work developed could have been introduced into the apparatus. Now, the only way in which this can have taken place is by the air having been colder, while it was expanding in the second process, than it was while it was undergoing compression during the first. Hence it was colder than 0° during the course of the second process; or, in other words, *while the water was freezing, under a pressure greater than that of the atmosphere, its temperature was lower than 0° .*

The fact of the lowering of the freezing point being thus demonstrated, it becomes desirable, in the next place, to find what is the freezing point of water for any given pressure. The most obvious way to determine this would be by direct experiment with freezing water. I have not, however, made any attempt to do so in this way. The variation to be appreciated is extremely small, so small, in fact, as to afford sufficient reason for its existence never having been observed by any experimenter. Even to detect its existence, much more to arrive at its exact amount by direct experiment, would require very delicate apparatus which would

* In saying "an equal quantity" I, of course, neglect infinitely small quantities in comparison to quantities not infinitely small.

not be easily planned out or procured. Another, and a better, mode of proceeding has, however, occurred to me: and by it we can deduce, from the known expansion of water in freezing, together with data founded on the experiments of REGNAULT on steam at the freezing point, a formula which gives the freezing point in terms of the pressure; and which may be applied for any pressure, from nothing up to many atmospheres. The following is the investigation of this formula:—

Let us suppose that we have a cylinder of the same imaginary construction as that of the one described at the commencement of this paper; and let us use it as an ice-engine analogous to the imaginary steam-engine conceived by CARNOT, and employed in his investigations. For this purpose, let the entire space enclosed within the cylinder by the piston be filled at first with as much ice as would, if melted, form rather more than a cubic foot of water, and let the ice be subject merely to one atmosphere of pressure, no force being applied to the piston. Now, let the following four processes, forming one complete stroke of the ice-engine be performed.

Process 1. Place the bottom of the cylinder in contact with an indefinite lake of water at 0° , and push down the piston. The effect of the motion of the piston is to convert ice at 0° into water at 0° , and to abstract from the lake at 0° the heat which becomes latent during this change. Continue the compression till one cubic foot of water is melted from ice.

Process 2. Remove the cylinder from the lake, and place it with its bottom on a stand which is a perfect non-conductor of heat. Push the piston a very little farther down, till the pressure inside is increased by any desired quantity which may be denoted, in pounds on the square foot, by p . During this motion of the piston, since the cylinder contains ice and water, the temperature of the mixture must vary with the pressure, being at any instant the freezing point which corresponds to the pressure at that instant. Let the temperature at the end of this process be denoted by $-t^\circ \text{ C}$.

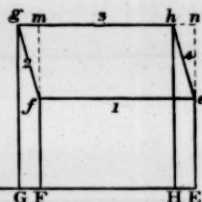
Process 3. Place the bottom of the cylinder in contact with a second indefinitely large lake at $-t^\circ$, and move the piston upwards. During this motion the pressure must remain constant at p above that of the atmosphere, the water in the cylinder increasing its volume by freezing, since, if it did not freeze, its pressure would diminish, and therefore its temperature would increase, which is impossible, since the whole mass of water and ice is constrained by the lake to remain at $-t^\circ$. Continue the motion till all the heat has been given out to the second lake at $-t^\circ$, which was taken in during Process 2, from the first lake at 0° .*

* This step, as well as the corresponding one in CARNOT's investigation, it must be observed, involves difficult questions, which cannot as yet be satisfactorily answered, regarding the possibility of the absolute formation or destruction of heat as an equivalent for the destruction or formation of other agencies, such as mechanical work; but, in taking it, I go on the almost universally adopted supposition of the perfect conservation of heat.

Process 4. Remove the cylinder from the lake at $-t^{\circ}$, and place its bottom again on the non-conducting stand. Move the piston back to the position it occupied at the commencement of Process 1. The temperature and pressure, during this process, must vary with one another, as they did in Process 2. Also, since as much heat has been given out as was taken in; and since the volume is the same as at the commencement of Process 1, the physical state of the mass contained in the cylinder must be now in every respect the same as it was at that time.

By representing graphically in a diagram the various volumes and corresponding pressures, at all the stages of the four processes which have just been laid down, we shall arrive, in a simple and easy manner, at the quantity of work which is developed in one complete stroke by the heat which is transferred during that stroke from the lake at 0° to the lake at $-t^{\circ}$. For this purpose, let E be the position of the piston at the beginning of Process 1; and let some distance, such as EG, represent its stroke in feet, its area being made a square foot, so that the numbers expressing, in feet, distances along EG may also express, in cubic feet, the changes in the contents of the cylinder produced by the motion of the piston.

Now, when 1.087 cubic feet of ice are melted, one cubic foot of water is formed. Hence, if EF be taken equal to .087 feet, F will be the position of the piston when one cubic foot of water has been melted from ice, that is, the position at the end of Process 1, the bottom of the cylinder being at a point A distant from F by rather more than a foot. Let ef be parallel to EF, and let Ee represent one atmosphere of pressure; that is, let the units of length for the vertical ordinates be taken such that the number of them in Ee may be equal to the number which expresses an atmosphere of pressure. Also let gh be parallel to EF, and let fm represent the increase of pressure produced during Process 2. Then the straight lines ef and gh will be the lines of pressure for Processes 1 and 2; and for the other two processes, the lines of pressure will be some curves which would extremely nearly coincide with the straight lines fg and he . For want of experimental data, the nature of these two curves cannot be precisely determined; but, for our present purpose, it is not necessary that they should be so, as we merely require to find the area of the figure $efgh$, which represents the work developed by the engine during one complete stroke, and this can readily be obtained with sufficient accuracy. For, even though we should



adopt a very large value for fm , the change of pressure during Process 2, still the changes of volume gm and hn in Process 2 and Process 4 would be extremely small compared to the expansion during the freezing of the water; and from this it follows evidently that the area of the figure $efgh$ is extremely nearly equal to that of the rectangle $efmn$, but fe is equal to FE , which is $\cdot 087$ feet. Hence the work developed during an entire stroke is $\cdot 087 \times p$ foot-pounds. Now this is developed by the descent from 0° to $-t^\circ$ of the quantity of heat necessary to melt a cubic foot of ice; that is, by 4925 thermic units, the unit being the quantity of heat required to raise a pound of water from 0° to 1° centigrade. Next we can obtain another expression for the same quantity of work; for, by the tables deduced in the preceding paper from the experiments of REGNAULT, we find that the quantity of work developed by one of the same thermic units descending through one degree about the freezing point, is $4\cdot 97$ foot-pounds. Hence, the work due to 4925 thermic units descending from 0° to $-t^\circ$ is $4925 \times 4\cdot 97 \times t$ foot-pounds. Putting this equal to the expression which was formerly obtained for the work due to the same quantity of heat falling through the same number of degrees, we obtain

$$4925 \times 4\cdot 97 \times t = \cdot 087 \times p.$$

Hence,

$$t = \cdot 00000355 p. \quad (1.)$$

This, then, is the desired formula for giving the freezing point $-t^\circ$ centigrade, which corresponds to a pressure exceeding that of the atmosphere by a quantity p , estimated in pounds on a square foot.

To put this result in another form, let us suppose water to be subjected to one additional atmosphere, and let it be required to find the freezing point. Here $p = \text{one atmosphere} = 2120$ pounds on a square foot; and, therefore, by

$$(1.) \quad t = \cdot 00000355 \times 2120.$$

or

$$t = \cdot 0075.$$

That is, the freezing point of water, under the pressure of one additional atmosphere, is $-\cdot 0075^\circ$ centigrade; and, hence, if the pressure above one atmosphere be now denoted in atmospheres,* as units by n , we obtain t , the lowering of the freezing point in degrees centigrade, by the following formula—

$$t = \cdot 0075 n. \quad (2.)$$

* The atmosphere is here taken as being the pressure of a column of mercury of 760 millimetres; that is 29.92, or very nearly 30 English inches.

XXXVIII.—*On the Gradual Production of Luminous Impressions on the Eye, and other Phenomena of Vision.* By WILLIAM SWAN, F.R.S.E.

(Read March 19, 1849.)

It is well known that a luminous object is seen for some time after its light has ceased to fall on the retina ; but less attention seems to have been paid to the fact, that light requires a certain time to produce its full impression on the eye. Accordingly, while it is stated in most treatises on optics, that the sensation of vision continues after the action of light has ceased, only a few writers have mentioned that the total effect of light on the eye is not produced instantaneously, but that a certain time is required for its complete development.

The merit of having first noticed this phenomenon of vision is probably due to Lord BACON, who observes, that notwithstanding the rapidity of the act of vision, a certain time is required for its exercise, which is proved by certain objects, such as a musket-ball, being invisible on account of the velocity of their motion. For the flight of the ball, he remarks, is too swift to allow an impression of its figure to be conveyed to the sight.*

While succeeding writers have devoted much attention to other departments of the physiology of vision, they have not, so far as I am aware, added a single fact to our knowledge of this part of the subject, which remains, therefore, precisely as it was left by Lord BACON.†

* "At in visu (cujus actio est perniciosissima) liquet etiam requiri ad eum actuandum momenta certa temporis : idque probatur ex iis, quae propter motus velocitatem non cernuntur ; ut ex latione pilae ex sclopeto. Velocior enim est praetervolatio pilae, quam impressio speciei ejus quae deferri poterat ad visum."—(*Novum Organum*, lib. ii., Aph. xlv. *Bacon's Works*, vol. i., p. 370. Lond. 1711.)

† This appears from the following passages, which will be found to contain little more than a repetition of Lord BACON's statement :—

"Il est un fait auquel on a généralement accordé peu d'attention, quoiqu'il ait été remarqué (voyez *Essai d'un Cours Élémentaire et Général des Sciences Physiques*, par M. BEUDANT : Partie Physique, p. 489 de la 3^{me} édition), c'est que les impressions directes exigent un certain temps pour se développer sur la rétine. Pour se convaincre de la réalité de ce fait, qui devait naturellement se prévoir a priori, il suffit de se rappeler qu'un objet qui passe très rapidement devant l'œil, ne se voit pas, ou s'aperçoit à peine. On peut encore prouver la chose par l'expérience suivante. Si l'on fait mouvoir circulairement, devant un fond noir, un petit morceau de papier blanc, avec une vitesse telle que l'anneau apparente qu'en résulte présente une teinte parfaitement uniforme et tranquille, cet anneau ne paraîtra pas blanc, mais gris. Or il suit de l'uniformité de la teinte, que pendant le petit intervalle de temps qui sépare deux passages successifs de l'objet au même point, l'impression ne décroît pas d'une quantité sensible : il faut donc nécessairement admettre que cette impression n'est pas blanche, comme celle qui est produite par l'objet en repos, mais qu'elle est grise, c'est-à-dire d'une blancheur imparfaite, ou enfin qu'à raison du temps extrêmement court que l'objet emploie à passer devant l'œil, il ne produit qu'une impression incomplète. Il est inutile d'ajouter qu'on obtiendra des résultats analogues en employant un objet d'une couleur quelconque : toujours l'anneau paraîtra plus sombre que l'objet en repos. L'éclat de l'anneau sera d'ailleurs d'autant moins éloigné de celui de l'objet en

Before I was aware that any one had noticed the gradual action of light on the eye, my attention was accidentally directed to that subject about eighteen months ago, by observing that the light of the sky seen immediately over a ball in its descent through the air, seemed less bright than at those parts of the retina where the action of the light had not been interrupted by the passage of the dark body. It immediately occurred to me, that this appearance was caused by the portion of the retina over which the image of the ball had passed, not having had time to be fully impressed with the light of the sky at the instant when the passage of the ball again exposed it to the action of that light.* Such an observation as this does not admit of easy repetition, but a more convenient method of exhibiting the gradual production of luminous impressions will be afterwards described.

It may be necessary here to anticipate an objection to the supposition, that light requires a sensible time to produce its full effect on the retina, founded on the observations of Professor WHEATSTONE, whose experiments prove, that "the light of electricity of high tension has a less duration than the millionth part of a second;" and that "the eye is capable of perceiving objects distinctly which are presented to it during the same small interval of time."†

It is obvious, however, that these statements are perfectly consistent with the gradual action of light on the eye. For, although light may produce a certain effect

repos, ou, en d'autres termes, l'impression approchera d'autant plus d'être complète, que cet objet aura plus de largeur, et que par suit il emploiera, dans son mouvement, un temps moins court à passer devant l'œil : ainsi l'expérience, que nous venons de décrire, conduit de plus à cette conséquence facile à prévoir que le développement de l'impression directe est progressive quoique très rapide."—(*Essai d'une Théorie Générale comprenant l'ensemble des Apparences Visuelles*, &c. par J. PLATEAU, p. 53. *Nouveaux Mémoires de l'Académie Royale des Sciences et Belles Lettres de Bruxelles*, tome viii., 1834.)

A statement almost identical with this will be found in PLATEAU *sur la Persistance des Impressions de la Rétine. Supplément au Traité de la Lumière* de Sir J. F. W. HERSCHEL. Par A. QUETELET. p. 474, 1833. See also MULLER's *Physics*, p. 274. London, 1847.

The following is the passage in BEUDANT *Cours de Physique*, to which M. PLATEAU refers:—"C'est aussi parce que l'impression d'un objet sur notre œil ne se fait pas instantanément, que nous ne pouvons apercevoir un corps qui se meut avec une extrême vitesse. Ainsi par exemple, un boulet de canon lancé par une bouche à feu, est invisible pendant une grande partie de son mouvement, parce qu'il ne reste pas assez de temps dans un même lieu, pour qu'on ait celui de l'apercevoir."

M. PLATEAU observes, in the passage which has just been quoted, that it was easy to foresee *a priori* that the development of the impression of light on the eye is progressive, although very rapid. With reference to this opinion, while it may be admitted that it is quite natural to suppose that the action of light on the eye is not absolutely instantaneous; yet, certainly, no one would be entitled to conclude *a priori* that a *sensible* time is required to produce impressions on the eye. I have, therefore, much satisfaction in availing myself of the present opportunity of directing attention to Lord BACON's prior claim to the merit of pointing out the curious and interesting fact, that light requires an appreciable time to produce visual impressions on the eye.

* It may be supposed that a different explanation of this effect might be afforded by the persistence of the impression of the image of the ball on the eye. That this explanation is identical with that given above, is evident from the image of the ball when seen projected upon the sky, being sensibly black. For, since blackness is the negation of light, the persistence of a black impression is but a want of light on that portion of the retina where the impression is perceived; and the existence of such an impression, or a want of luminosity after the eye is fully exposed to light, clearly proves that its action on that organ is not instantaneous.

† Philosophical Transactions, 1834, p. 591.

on that organ in the millionth part of a second, it by no means follows that this is its full effect; and thus, while the electric spark renders objects distinctly visible which are seen for less than the millionth part of a second, it may still be true that the apparent brightness of those objects would increase if the duration of the light could be prolonged.

Having found only very brief and general references, to the gradual action of light on the eye, in any authors to whose works I have had access, I resolved to investigate the subject experimentally; and the object of this paper is to describe a series of experiments undertaken for the purpose of ascertaining the connexion between the brightness of the impression produced by light on the retina, and the time during which it acts on the eye.*

Before entering upon the narrative of my experiments it may be proper to premise, that if, in some cases, I seem to assume that the results obtained by experiments on my own eyes, are to be regarded as universal phenomena, I do so merely to avoid circumlocution; and I believe I may plead the example of most writers who describe experiments on vision in justification of such an apparent assumption.

In several cases, however, some of which will be afterwards noticed, the experiments have been witnessed by others, whose concurrent testimony has proved that the results were not dependent upon any idiosyncrasy of vision on my part.

I may also have made use of expressions which seem to involve the assumption that the brightness with which a luminous object is seen at any instant, is the same as the apparent brightness of its image on the retina at that instant: or, in other words, that the impression of light on the retina is perceived by the mind instantaneously. Such expressions, however, are employed simply for the sake of brevity. The principal object of my investigation is to determine the brightness of an impression made on the retina by a light of a given intensity, acting for a given time; and it will be found in the sequel that the method I have devised for measuring the time during which the light acts, and the intensity of the resulting impression, does not depend for its accuracy on the settlement of the question, whether or not the impressions of light on the retina are instantaneously perceived by the mind.

I. *Method of Observation.*

In order to examine the phenomena presented by luminous impressions of short duration, I made use of the following method of observation. If a disc of

* Additional proof that almost no attention has hitherto been paid to this subject, may be derived from the fact, that no notice is taken of it in MULLER's *Physiology*, London 1839, nor in the Supplement to that work by BALY, London 1848. M. PLATEAU also observes: "Personne n'a essayé de mesurer le temps nécessaire à la production complète de l'impression." PLATEAU *Sur la Persistance des Impressions*. (*Supplément au traité de la Lumière de Sir J. F. W. HERSCHEL*. Par A. QUETELET, p. 474. 1833.)

pasteboard (see Fig. 4, Plate XII.), or other convenient material, having a portion of a sector $ABCD$, cut out from its circumference, be made to revolve, in a plane perpendicular to the line of vision, between the eye and a luminous object, the object may be placed so as to be seen through the sector at each revolution of the disc. In this manner a succession of luminous impressions will be obtained; and the time during which the light acts on the eye at each impression will depend partly on the velocity of the rotation of the disc, and partly on the ratio of the arc of the sector to the whole circumference.

Let ABG (Fig. 1) represent the disc, ACB the sector cut out of it, and ED the section, by the plane of the disc, of the pencil of rays proceeding from the luminous object to the eye. Then, if θ = the angle ACB , and t = the time in which the disc makes one revolution; the time in which the line AC revolves from its present position to the position BC will evidently be $\frac{t\theta}{2\pi}$.

Now, if a ray proceeding from any point in the luminous surface is just emerging at F , the point from which it emanates will remain visible until AC comes to the position BC , or during the time $\frac{t\theta}{2\pi}$. Since this is obviously true of any other element of the surface, it follows that every part of the surface remains visible for the same time.

The interval of time between the first appearance of the object and its final disappearance is obviously greater than that during which each element of its surface is visible. For, if E and D be sections of the rays proceeding from the points in the luminous surface which are first and last visible, some part of the surface will be seen during the interval of time between the instant in which CB coincides with CE , and that in which AC coincides with CD , or during the time in which the line AC revolves through the sum of the angles ECD , ACB . Denoting ECD by λ , this time will be $\frac{t(\theta + \lambda)}{2\pi}$.

If the luminous object is circular, and the axis of the pencil of rays proceeding to the eye is perpendicular to the plane of the disc, putting

s = the radius of the luminous circle,

d = its distance from the eye,

d' = the distance of the disc from the eye,

c = the distance of the axis of the pencil of rays from the centre

of the disc, it will be found that $\lambda = 2 \sin^{-1} \frac{s d'}{c d}$; and therefore the time which elapses between the first appearance and the final disappearance of the luminous circle, is

$$\frac{t}{2\pi} \left(2 \sin^{-1} \frac{s d'}{c d} + \theta \right)$$

From this expression it will be seen that the time during which the eye re-

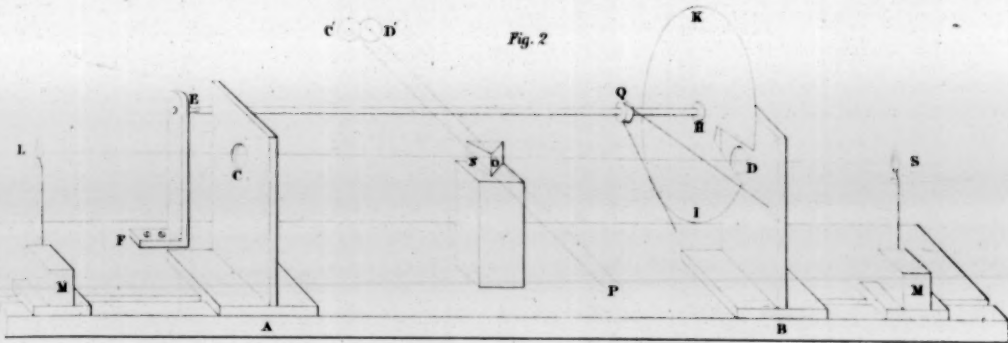


Fig. 2

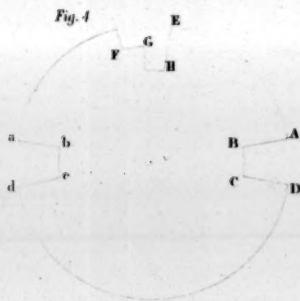


Fig. 4



Fig. 5

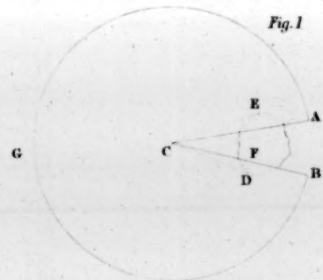
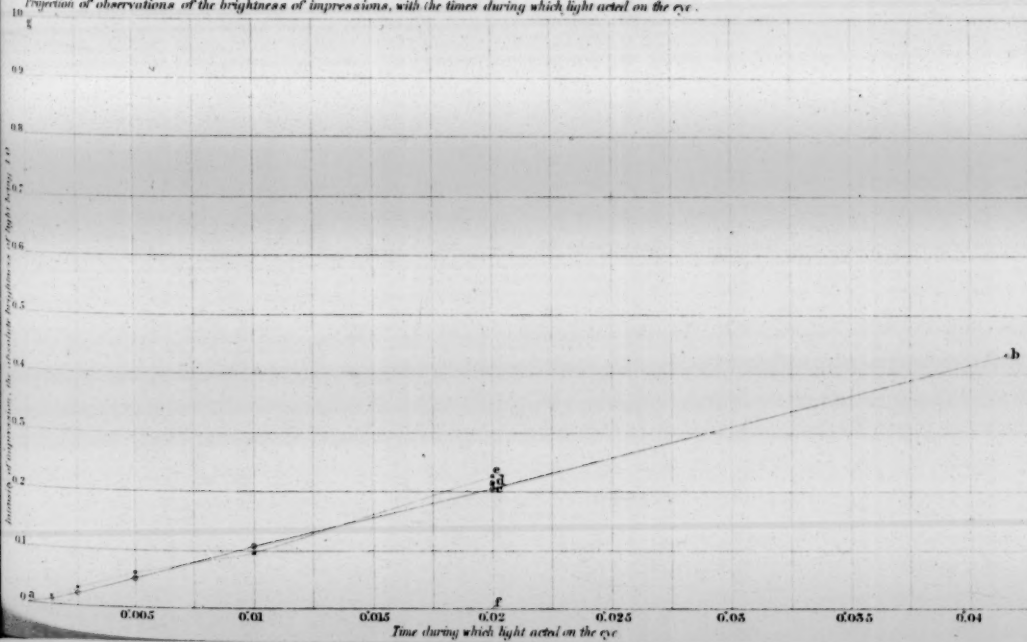


Fig. 1

Fig. 3

Projection of observations of the brightness of impressions, with the times during which light acted on the eye.





ceives light at each revolution of the disc may be varied, by altering the diameter of the luminous circle, or its distance from the eye, or from the disc, or by changing the distance of the pencil of transmitted rays from the centre of the disc. But in the experiments to be afterwards described, these elements remained constant, and the effect was modified only by altering the angles of the sectors, or the rate of revolution of the discs. It may, however, be proper to observe that, as the time during which each element of the surface remains visible is independent of the magnitude of the luminous object; so also, as might be anticipated, the apparent brightness of the surface is independent of the ratio which the portion visible at once through the sector bears to the whole area. For, in repeating the same experiments with circular luminous objects of different diameters, while the angle of the sector and the velocity of the disc were constant, it was found that the apparent brightness of the luminous circle was not sensibly affected by varying its diameter.

In order to compare the brightness of the impressions produced by light seen through the sectors of revolving discs, in the manner now described, with its brightness when seen by uninterrupted vision, the following arrangement was devised, which, for the sake of convenient reference, may be termed a *selaometer* (from *σελας*, *brightness*), to indicate its use in measuring the brightness of luminous impressions. This apparatus, represented in Fig. 2, is supported on a stout plank A B. A C and B D are screens with circular apertures C, D, an inch in diameter, to which are fitted pieces of ground glass, cut from the same plate, in order to secure similarity of surface. The apertures C, D, are illuminated by the gas-burners, L, S,* which are supplied by flexible tubes, so that their distances from the screens can be varied at pleasure by sliding their supports M, M, along the plank A B, in a groove cut in it for that purpose. An axis E H, carrying the disc I K, revolves between conical points in supports, one of which E F is seen in the figure. This axis is put in motion by a band passing over the pulley Q, and over a wheel driven by means of a winch; and it is made of sufficient length to admit of a second disc revolving in front of the screen A C simultaneously with the disc I K. The brightness of the apertures in the screens is observed by means of a rectangular prism of glass N O, placed half-way between them, with its faces inclined at angles of 45° to the line C D joining their centres. By this means the light passing through the apertures, and dispersed by the ground-glass, is reflected from the faces of the prism to the eye at P, and the images of the apertures being seen in apparent contact, as represented at C', D', their relative brightness can be compared with great nicety.† The driving-wheel is made

* The gas-light used in all the experiments described in this paper, was that of coal-gas burned by a No. 2 swallow-tail jet. It will be seen that the numerical results, afterwards obtained, do not depend on the absolute brightness of this light.

† Screens covered with black paper, which are not represented in the figure, were used to protect the eyes from the action of extraneous light, and also to intercept any rays, whose influence might have otherwise affected the accuracy of the experiments.

to revolve in time with a metronome adjusted to beat seconds; so that, by ascertaining the number of revolutions which the pulley, Q, makes during each revolution of the driving-wheel, the time of a single revolution of the disc is readily determined. This time multiplied by the ratio of the arc of the sector to the whole circumference of the disc, gives the length of each luminous impression. Thus, if the driving-wheel revolves m times in a second, and the disc n times during each revolution of the driving-wheel, the time of revolution of the disc, expressed in seconds, is $\frac{1}{mn}$; and if θ be the angle of the sector, the time during which the eye receives light from each element of the luminous surface at every revolution of the disc is $\frac{\theta}{2mn\pi}$.

In order to compare the brightness of the aperture D, seen by uninterrupted vision, with its brightness as seen during the revolution of the disc, the illumination of the apertures is first made equal by varying the distance of the flame L from the screen AC, until both apertures seen by reflexion in the prism appear equally bright. When the disc is then made to revolve, the apparent brightness of the aperture D immediately diminishes, and the equality of the brightness of the apertures is again restored by withdrawing the light L, to a greater distance from the screen AC.* Since the distance of the light S, from the screen BD, remains constant during this operation, the ratio of the apparent brightness of the aperture D, seen by uninterrupted vision, to its apparent brightness during the revolution of the disc, will be that of the square of the distance of the light L, from the screen AC, before the disc has begun to revolve, to the square of its distance during the revolution of the disc. For since the intensity of the light incident on the face O, of the prism is constant, we may conceive that face of the prism as the source of light of a constant intensity. Let b_1 = the apparent brightness of this light seen by uninterrupted vision; b_2 = its apparent brightness seen during the revolution of the disc. Then if i = the intrinsic brightness of the flame L, d_1 and d_2 its distances from the screen before and during the revolution of the disc, a , the ratio of the brightness of the light transmitted by the glass in the aperture C', to that incident upon it, and r , the ratio of the brightness of the reflected light to the light incident upon the face N, of the prism; the apparent brightness of the aperture C, when the light L, is at the distance d_1 , will be $\frac{ari}{d_1^2}$, and at the distance d_2 its apparent brightness will be $\frac{ari}{d_2^2}$. Now since the apparent brightness of both apertures is made equal, we have

* This is conveniently done by means of a pulley and cord. When the apertures are being made equally bright before the disc is made to revolve, it is necessary that the aperture D should be fully exposed. Where the sector is too narrow to admit of the whole aperture being seen at once, another sector is cut in the disc for this purpose, which admits of being closed by a slider of pasteboard before the disc is made to revolve.

$$\frac{b_1}{b_2} = \frac{\frac{a r i}{d_1^2}}{\frac{a r i}{d_2^2}} = \frac{d_2^2}{d_1^2}$$

It has here been assumed that the brightness of the gas-flame remains constant during the experiment, a condition which is not fulfilled in practice, owing to the variable pressure of the gas in the pipes. It is probable, however, that the brightness of both flames will vary nearly in the same proportion, so that the distances necessary to equalise the apparent brightness of the apertures in the screens will remain almost unaltered; and it is obvious also, that any residual error, arising from a gradual change in the brightness of the flames, will be nearly eliminated by taking the mean of a series of observations immediately succeeding each other, and conducted in the manner now described.

II. Proof of the Gradual Action of Light on the Eye.

If a disc, with a sector of a small angle, is made to revolve between the eye and a luminous object, a flash of light is seen at each revolution; but as the velocity of rotation increases, the brightness of the flashes diminishes, which shews that the apparent brightness of a luminous object diminishes as the time during which it is visible becomes shorter. A similar result is obtained by placing two discs with sectors of different angles before the screens of the selascope, and observing the relative intensity of the simultaneous flashes of light when the discs are made to revolve. It will always be found that although the apertures in the screens are equally bright, when seen by uninterrupted vision, the disc whose sector has the greater angle produces the brighter flash. Now as both discs revolve with the same velocity, the length of the luminous impressions will be proportional to the angles of the sectors; so that, by this experiment also, the apparent brightness of the light is shewn to increase with the time during which it continues to act upon the eye. A convenient *selascope*, which exhibits this phenomenon in a striking manner may be made by causing a disc with a sector of the form E F G H (Fig. 4), to revolve before a luminous aperture. The flash produced by the wide part of the sector E F, greatly exceeds in brightness that produced by the narrow part G H.

The experiment of causing a disc, with a sector cut in it, to revolve before a luminous aperture, also affords a simple proof of the duration of luminous impressions on the retina. At about seven revolutions in a second the luminous impression becomes continuous, so that the aperture always appears visible, even during the dark interval between the successive passages of the sector between it and the eye; but a considerably higher velocity, about twenty revolutions in a second, is required in order to produce a sensibly uniform impression. These velocities are not meant to be stated with great exactness. They were obtained by experi-

ments with discs having sectors varying from 60° to $7^\circ 30'$, and did not appear to differ very sensibly in different cases; but it seems probable that the velocity required to produce a continuous or a uniform impression should be sensibly affected by greatly altering the angle of the sector.

III. *Effect of combined Luminous Impressions on the Eye.*

So long as the rotation of the disc is so slow as to allow each flash to be seen separately, the brightness of the flashes diminishes as the velocity of rotation increases, until, at about twenty revolutions in a second, the flashes become blended into a nearly uniform impression. Whenever this takes place, no farther increase of the velocity of the disc diminishes the intensity of the impression in the smallest perceptible degree. This result is evidently produced by the increased number of luminous impressions in a given time compensating for their diminished intensity; but it is remarkable that the one effect should so exactly compensate for the other. Having found that this compensation took place at velocities varying from twenty to forty revolutions in a second, I was anxious to ascertain whether it continued unimpaired at higher velocities. For this purpose a disc of pasteboard 4.5 inches in diameter, with a sector of $2^\circ 30'$ cut out of its margin, was fitted to the axle of a clockmaker's wheel-cutting engine. It was found by a previous careful trial, that the disc made exactly 100 revolutions for each revolution of the driving-wheel; and as the latter, at its greatest velocity, made thirteen revolutions in ten seconds, the disc ought to have revolved 130 times in a second. But to avoid the chance of errors arising from the driving-bands slipping at so high a velocity, I availed myself of Professor WHEATSTONE's ingenious method of ascertaining the velocity of a rapidly-revolving axle, described in his paper in the *Philosophical Transactions* for 1834, to which I have already referred. This consisted in observing the pitch of the note produced by the rapid percussion of a pin, fixed in the revolving axle, upon a piece of paper held in contact with it. The highest note produced during the experiment was rather less than an octave below C of the tenor clef, which corresponds to above 128 vibrations in a second. This result agrees almost exactly with the calculation founded on the observed rotation of the disc at low velocities; and it may, therefore, be concluded, that the disc made above 128 revolutions in a second. Since the arc of the sector was $\frac{1}{144}$ of the circumference, the light from a luminous point placed behind the disc would, at each revolution, act on the eye for only $\frac{1}{18432}$ of a second. A lighted candle being placed behind the disc, the machine was put in motion, and the velocity gradually increased until the driving-wheel made thirteen revolutions in ten seconds, after which it was allowed to come to rest spontaneously. It was found that the brightness of each successive flash diminished as the velocity of the disc increased, until the impression on the eye became uniform, at a velocity of

about twenty revolutions in a second. After this no increase of velocity, up to 128 revolutions in a second, produced the slightest farther diminution of the apparent brightness of the light; and again, as the speed diminished the light continued uniformly bright, until the motion became so slow as to allow the eye to perceive the impressions separately, after which they gradually increased in intensity until the disc stopped. The same experiment was repeated, substituting for the flame of the candle an illuminated aperture in a screen, covered with tissue paper. The apparent brightness of this aperture, when the disc revolved, was compared, in the manner already described at p. 585, with that of another similar aperture seen by uninterrupted vision, and the result was perfectly in accordance with that obtained in the previous experiment. The same phenomena were also observed when a disc with a sector of 30° was substituted for that with a sector of $2^\circ 30'$.*

A similar result was obtained by varying the form of the experiment, in the following manner:—Two discs, one with a sector of 30° , and the other with two sectors of 15° , such as A B C D, *a b c d* (Fig. 4), at opposite extremities of its diameter, were placed in the selaometer, and made to revolve simultaneously. In both discs the ratio of the duration of the flashes to that of the dark intervals, is obviously the same; but when the discs revolve simultaneously, for each flash produced by the disc with the sectors of 30° , there are two flashes of half the duration produced by the disc with two sectors of 15° . The disc with two sectors of 15° , revolving at a given velocity, is, therefore, precisely identical in its action to the disc with a single sector of 30° revolving at double the velocity. The apertures in the screen being made equally bright before the discs revolved, the equality of their brightness remained unaltered when the discs revolved so rapidly as to produce a uniform impression upon the eye. In the same manner, the brightness of the apertures remained equal when any disc, in the first part of the following table, revolved simultaneously with the corresponding disc in the second part, at such a velocity as to produce a uniform impression on the eye.

Number of Sectors.†	Angle of Sectors.	Number of Sectors.	Angle of Sectors.
2	15°	1	30°
4	$7^\circ 30'$	1	30°
2	$7^\circ 30'$	1	15°
2	30°	1	60°
3	30°	1	90°
4	30°	1	120°

In all these cases, the duration of each flash was inversely as the number of

* I was enabled to make this experiment by the kindness of Mr ALEXANDER BRYSON, who, along with Mr JOHN TURNBULL, W.S., witnessed the results above described.

† In all experiments in which the discs had more than one sector, the sectors were arranged round the circumference at equal distances from each other.

flashes in a given time; and these experiments, therefore, confirm the result obtained by varying the velocity of a disc with a sector of a given angle. We may, therefore, infer,

1st, That if the number of flashes, in a given time, succeeding each other so rapidly as to produce a uniform impression on the eye is inversely as the duration of each flash, their aggregate effect on the eye will be constant.

2dly, This compensation of the diminished intensity, by the increased frequency of the flashes, is independent of the interval of time between each impression, within the limits of the observations; that is, with intervals varying from $\frac{1}{20}$ th to $\frac{1}{128}$ th of a second.

3dly, The effect is also independent of the ratio of the duration of the luminous to that of the dark intervals.

It is thus shewn that, after a uniform impression is produced, increasing the number of flashes in a given time, compensates for their diminished intensity. This naturally leads to the inquiry, at what rate would the brightness of the resulting impression increase with the number of flashes in a given time, supposing the intensity of the flashes to remain constant? In order to ascertain the connexion between the number of flashes of a given intensity in a given time, and the intensity of their combined effect on the eye, I made the following experiments:—

1. Two discs, one with a sector of 15° , and the other with two equidistant sectors of 15° , as A B C D, *a b c d*, Fig. 4, were placed in the selaometer, and the screens were equally illuminated by carefully adjusting the distances of two similar spermaceti candles placed behind them. When the discs were made to revolve so rapidly as to produce a uniform impression, a second candle placed behind the screen, whose disc had a single sector of 15° , restored the equality of the apparent brightness of the apertures, and a similar result was obtained when discs with single sectors of 30° and $7^\circ 30'$, were compared with discs having two sectors of the same angles.

2. To vary the experiment, the flame of a gas burner was placed 10 inches behind the screen, whose disc had two sectors of 30° , and a similar flame was adjusted behind the screen, whose disc had a single sector of 30° , so as to illuminate the apertures in the screens equally. When the discs revolved rapidly, the flame behind the disc with two sectors was withdrawn to 14.1 inches ($10\sqrt{2}$), so as to halve the intensity of the light incident on the screen, and the illumination of the apertures appeared to be perfectly equal. In like manner, when the light was first placed at 20 inches from the screen, after the discs revolved, the screens seemed equally illuminated when it was withdrawn to 28.2 inches.

3. A disc with a single sector of $7^\circ 30'$, and another with two sectors of the same angle, were placed in the selaometer. The illumination of the screens was then made equal by adjusting the distance of the light behind the disc with two

sectors. The distance of this light was noted, and, when the discs revolved rapidly, the light was withdrawn until the apertures seemed again equally bright. The degree in which the brightness of the light required to be diminished, by withdrawing the flame from the screen, in order to equalise the brightness of the impressions produced by the two discs, was taken as a measure of the ratio of the brightness of those impressions when the screens were equally illuminated. Four experiments were made, and, as formerly, putting d_1 and d_2 to denote the distances of the light from the screen before and after the disc had been made to revolve, and ρ to denote the ratio of the apparent brightness of the apertures seen during the revolution of the discs when their illumination was actually equal, the mean values of those quantities was found to be

$$d_1 = 13.82; d_2 = 20.13; \rho = 2.122.$$

In a second set of four experiments,

$$d_1 = 4.075; d_2 = 5.95; \rho = 2.132.$$

4. When a disc with three equidistant sectors of $7^\circ 30'$ was compared with a disc having a single sector of $7^\circ 30'$, the mean of four experiments gave

$$d_1 = 14.15; d_2 = 24.175; \rho = 2.920.$$

In other four experiments,

$$d_1 = 4.1; d_2 = 7.15; \rho = 3.041.$$

5. A disc with two sectors of 30° , compared four times with a disc having a single sector of 30° , gave

$$d_1 = 4.05; d_2 = 6.1; \rho = 2.269.$$

and a second set of four experiments,

$$d_1 = 13.95; d_2 = 20; \rho = 2.056.$$

6. A disc with four sectors of 30° , compared four times with a disc having a single sector of the same angle, gave

$$d_1 = 3.95; d_2 = 7.925; \rho = 4.026,$$

and a second trial of four experiments,

$$d_1 = 14.1; d_2 = 29.4; \rho = 4.348.$$

In all these experiments, the discs revolved so rapidly, as to produce a uniform impression on the eye, but the equality of illumination, when once obtained, was not affected by increasing the velocity; and from the variation of the quantity d_1 in the different experiments, it will be seen that the results are independent of the intrinsic brightness of the light incident on the screen.

The following table exhibits the mean values deduced from experiments No. 3 to No. 6 inclusive, and shews that the brightness of the impression produced by rapidly succeeding flashes of light of a given intensity, is sensibly proportional to the number of flashes in a given time.

Number of Flashes in a Second.	Brightness of Impression.
20	1.000
40	2.141
60	2.980
80	4.184

In these experiments it is assumed, that when the light is withdrawn from the screen, so as to diminish the intensity of its illumination, the brightness of a flash of short duration will be diminished in the same ratio. This will only be true, provided lights of different intensity produce impressions of proportional intensity in equal times; but if it be afterwards proved, independently of these observations, that such is really the case, the conclusion which has now been drawn from the experiments will be perfectly correct. In order to avoid this assumption, the brightness of the impressions produced by the revolution of the different discs was next compared with the impression of uninterrupted light.

1. A disc, with a single sector of 30° , was placed in the selaometer, with a gas-flame at a constant distance of 6 inches from its screen. In order to render the illumination of the screens equal, the other light had to be placed at a distance of 5.9 inches from the second screen. When the disc revolved 20 times in a second, the latter light was gradually withdrawn until the apertures in the screens again appeared equally bright. The distance of that light was now found to be 22.2 inches. This experiment, eight times repeated, gave the following mean values:—

$$d_1 = 5.78; d_2 = 22.21; b_1 = 0.06773.$$

where b_1 denotes the brightness of the impression produced during the revolution of the disc. In like manner, a disc, with two sectors of 30° , treated in precisely the same manner, gave the following mean values of eight trials:—

$$d_1 = 5.61; d_2 = 15.22; b_2 = 0.1359.$$

b_2 denoting the brightness of the impression produced by the revolution of this disc. From this experiment, the ratio of the brightness of the impressions produced by the revolution of the two discs, or $\frac{b_2}{b_1}$ will be found to be 2.006.

2. A disc, with three sectors of 30° , was compared with direct light in the same manner. The mean of eight experiments gave

$$d_1 = 5.61; d_2 = 12.11; b_3 = 0.2164.$$

From which the ratio of the brightness of the impression produced by means of this disc, with three sectors, to that produced by means of the disc with a single sector, or $\frac{b_3}{b_1}$, is 3.169.

3. The mean of a similar set of experiments made with two discs, one having a single sector, and the other two sectors of 15° , gave $\frac{b_2}{b_1} = 2.099$.

4. Two discs, one having a single sector, and the other two sectors of $7^\circ 30'$ were compared, and it was found that $\frac{b_2}{b_1} = 1.851$.

The results of the experiments in the last four sections are shewn in the accompanying Table.

Number of Flashes in a Second.	Brightness of Impres- sion.
20	1.000
40	2.006
40	1.851
60	3.169

In this table, as in the last, the corresponding numbers in the opposite columns will be seen to be almost exactly proportional; and both sets of experiments therefore, lead to the following results:—

1. The brightness of the impression produced by equal flashes of light, which succeed each other so rapidly as to produce a uniform impression on the eye, is exactly proportional to the number of flashes in a given time.

2. Within the limits of the different velocities of the discs in the experiments, the effect of the combination of the flashes is not sensibly affected by the length of the dark intervals between them.

3. With the same limitation, the effect is also independent of the time of duration of the flashes.

IV. *On the connexion between the apparent Brightness of Light and the time during which it continues to act on the Eye.*

It has thus been proved that the brightness of the impression produced by rapidly succeeding flashes of light is proportional to the number of flashes in a given time, provided the brightness of the flashes remains constant. Hence, if a rapidly revolving disc, with a sector of a given angle, has its velocity doubled, and, consequently, the number of flashes produced by it in a given time also doubled, if the brightness of the flashes remains unaltered, the brightness of the impression produced by them will be twice as great as at first. But, instead of the brightness of the impression increasing, it has been found to continue unchanged, notwithstanding the increased velocity. It is, therefore, evident that when the velocity of the disc is doubled, and, consequently, the duration of each flash is half as great as at first, its brightness is also half as great as at first. Thus, if the disc first revolves 20 times in a second, and then 40 times in a second,

the intensity of the impression is precisely the same in both cases. But at the velocity of 40 revolutions in a second, there are twice as many flashes in a given time as there are at the first velocity; and if the brightness of the flashes was the same as at the first velocity, the brightness of the impression produced by them would be doubled. Since, therefore, the impression, instead of being doubly bright, remains the same as at first, each flash at 40 revolutions in a second must only be half as bright as at 20 revolutions in a second. In like manner, by supposing the velocity increased to 80 revolutions in a second, it might be shewn that the brightness of the flashes is again halved. But the effect of doubling the velocity is to halve the duration of the flashes, therefore the brightness of the flashes is proportional to their duration. This law of vision may be thus stated: When light of a given intensity acts on the eye for a short space of time, the apparent brightness of the luminous impression on the retina is exactly proportional to the time during which the light continues to act. From the velocities of the discs, and the angles of the sectors used in the experiments, it will be seen that this law is true for impressions lasting from $\frac{1}{18432}$ to $\frac{1}{120}$ of a second; and it will presently be shewn to be true for impressions of longer duration.

V. Observations of the apparent Brightness of Luminous Impressions of short duration.

In almost all the experiments hitherto described, the phenomena of vision which have been investigated have been derived from the observation of the aggregate effect of luminous impressions succeeding each other so rapidly as to produce a continuous impression on the eye. It is obvious, that such experiments afford no information regarding the absolute brightness of the separate impressions which are thus blended together. I adopted the indirect mode I have now described of ascertaining the connexion between the duration and apparent brightness of luminous impressions, from an apprehension of the difficulty of comparing the brightness of a constant light with that of an isolated flash. But repeated trials satisfied me that my fears were groundless; and the succeeding experiments prove that, with a little practice, the eye is perfectly capable of making this comparison. Such experiments cannot, however, be long continued without fatiguing the eye, and a considerable effort of attention is required for their successful performance.

In order to find the intensity of separate impressions of short duration, I used a disc of wood two feet in diameter, revolving once in a second; so that a sector, whose arc had a known ratio to the circumference of the disc, passed at each revolution before the aperture in one of the screens of the selsaometer. In this manner, a series of perfectly isolated impressions was obtained; and the intensity of each could be compared with that of a light seen by continuous vision in the manner already described. The different sectors were cut in paste-board, and placed over an aperture in the disc. The following experiments were made:—

1. The fixed light was placed six inches behind the screen before which the disc revolved. The sector had an angle of $0^\circ 27'$, or $\frac{1}{800}$ of the circumference of the disc; and the disc revolved once in a second. To equalise the brightness of the apertures in the screens, when both were seen by continuous vision, the light behind the second screen was placed at a distance (d_1) of 5.3 inches. But, when the disc revolved, this light had to be withdrawn to a distance (d_2) of 46.3 inches. This experiment was repeated ten times, with the following mean results: $d_1=5.11$; $d_2=50.29$; and the brightness of the flashes $b=0.0103$, the brightness of the light seen by continuous vision, being unity.

2. A sector of $0^\circ 54'$, or $\frac{1}{400}$ of the circumference, was next used, and ten experiments made as before, from which

$$d_1=5.23; d_2=35.24 \text{ and } b=0.022.$$

3. With a sector of $1^\circ 48'$, or $\frac{1}{200}$ of the circumference,

$$d_1=5.19; d_2=23.52; b=0.0487.$$

4. With a sector of $3^\circ 36'$, or $\frac{1}{100}$ of the circumference,

$$d_1=4.9; d_2=15.08; b=0.1056.$$

5. With a sector of $7^\circ 12'$, or $\frac{1}{50}$ of the circumference,

$$d_1=5.13; d_2=11.3; b=0.2061.$$

6. With a sector of 15° , or $\frac{1}{25}$ of the circumference,

$$d_1=5.02; d_2=7.64; b=0.4317.$$

With each sector, the mean of ten results was taken; and at each successive trial, the flame was alternately drawn from the screen, or pulled towards it in equalising the apparent brightness of the apertures in the screens. The following Table contains the results of these experiments, the brightness of the light seen by continuous vision being expressed by unity:—

d_1	d_2	Duration of Flash in Seconds.	Brightness of Impression.
5.11	50.29	0.00125	0.0103
5.23	35.24	0.00250	0.0220
5.19	23.52	0.00500	0.0487
4.90	15.08	0.01000	0.1056
5.13	11.30	0.02000	0.2061
5.02	7.64	0.04167	0.4317

The results of these experiments are shewn in fig. 3, where the observed intensities of the light, denoted by small circles, are projected with the corresponding times during which it acted on the eye; and it will be observed, that the line acb , shewing the increase of the apparent brightness of the object, with the time during which it remains visible, is very nearly straight; which proves that

within the limits of the observations the brightness of the light increases in exact arithmetical proportion with the time during which it acts on the eye. Since the observed intensities of the lights when projected, as in the figure, are all nearly included in a straight line passing through the origin, it may naturally be inferred, that the impression of light commences at the instant of its incidence on the retina. This conclusion is strengthened, when it is recollected that the preceding experiments prove that light, which is incident on the eye only $\frac{1}{18432}$ of a second, produces a distinct impression, while, according to Professor WHEATSTONE, less than the millionth part of a second is necessary for this effect. It has also been proved (see p. 594), that up to $\frac{1}{18432}$ of a second, the impression produced by light is proportional to its duration. It seems, therefore, highly probable, that from 0" up to 0"·05, *the brightness of a luminous impression is exactly proportional to the time during which the light has acted on the eye.**

These experiments, therefore, confirm, in a very satisfactory manner, the inference which has already been drawn from the previous investigation, as the observed intensities of the flashes are very nearly proportional to their duration: while, at the same time, they exhibit the actual numerical ratio of the apparent brightness of a flash of a certain duration, to that of the light which produces it acting continuously on the eye.

VI. *The time required for the complete production of Luminous Impressions is independent of the apparent intrinsic brightness of the light.*

The following series of experiments was made partly to confirm the result already obtained; but more especially in order to ascertain whether the time required for the complete development of luminous impressions varies with the brightness of the light by which they are produced. In this set of experiments, the same sectors were used as in the last; and the circumstances were identical, except that the fixed light was placed 8·5 inches ($6\sqrt{2}$) from the screen, so that the brightness of the incident light was reduced to half its former intensity. The following Table exhibits the mean of ten observations with each sector; the brightness of the light seen by continuous vision, being expressed by unity.

d_1	d_2	Duration of Flash.	Brightness of Flash.
7·32	64·17	0·00125	0·0130
7·31	44·12	0·00250	0·0275
7·56	33·60	0·00500	0·0508
7·62	24·20	0·01000	0·0991
7·60	16·06	0·02000	0·2240

* In an experiment made since this paper was read, I have found that the same law extends to impressions lasting for $\frac{1}{16}$ th of a second, of which the observed brightness was 0·6118.

In fig. 3, the line ae contains the projections of these observations, which are denoted by crosses; and it nearly coincides with the line ab , containing the projections of the observations in the last Table, shewing that the ratio of the brightness of an impression of given duration to that of the absolute brightness of the light which produces it, is almost exactly the same in both sets of experiments. On thus comparing the apparent intensities of the flashes exhibited in the above Table with the similar results in the preceding one, it will be seen that although the absolute intensity of the light is only half as great as formerly, the time required for the propagation of the luminous impression on the eye remains unaltered; while both sets of experiments prove that the brightness of a luminous impression caused by a light of given intensity is proportional to the time during which the light acts on the eye.

On repeating the experiment with the sector of $\frac{1}{50}$ th of the circumference, revolving once in a second, with the fixed light 24 inches from the screen, the mean of ten trials gave

$$d_1 = 22.27; d_2 = 48.46; b = 0.2112.$$

The ratio of the apparent brightness of the flashes to that of the light seen by continuous vision is, in this case, almost exactly the same as in the preceding experiments, as will be seen from the following comparative view:—

Distance of Light.	Intensity of Light.	Time during which Light acted on the Eye.	Ratio of the brightness of Flashes to that of the Light seen continuously.
6.0	1.0000	0.02	0.2061
8.5	0.5000	0.02	0.2240
24.0	0.0625	0.02	0.2112

The conclusion to be derived from these results will be distinctly apprehended by reference to fig. 3, where the ordinates ef , cf , and df , represent the apparent intensities of the lights shewn in the above table. In order to prevent misunderstanding, it is necessary to observe, that although the absolute brightness of the lights used in the three experiments given in the table are in the ratio of the numbers 1, 2, and 16, they are all represented in the figure by the same line ag ; and since the lines ef , cf , df , are nearly equal, they may be regarded as having the same ratio to ag , the slight differences between them obviously resulting from errors of observation. It thus appears that after an interval of $\frac{1}{50}$ th of a second, the three lights of very different intensity have all produced the same portion of their total effect on the eye; the impression in each case having nearly $\frac{1}{5}$ th of the absolute brightness of the light.

Lights of different intensity, therefore, produce like portions of their total effect

on the eye in equal times ; from which it obviously follows, that the brightness of an impression on the eye increases with a rapidity exactly proportional to the brightness of the light which produces it.

This conclusion seemed so remarkable, that I determined to try whether the direct light of the sun produced a given portion of its impression on the eye with no greater rapidity than ordinary artificial light. For this purpose I made use of a sclaometer, represented in fig. 5, where K L represents a plate of brass with two apertures A B, $\frac{1}{8}$ th of an inch in diameter, and half an inch distant. A plate of ground glass is placed before the apertures, and behind the aperture B, a tube B C is fixed, in which is placed a Nicol's polarizing prism. A longer tube B D, is fitted so as to turn freely upon the outside of the tube C D, and another Nicol's prism is placed in its further extremity, so that, by turning round the tube B D, the illumination of the aperture B can be varied at pleasure. A disc E F, with a sector of $7^{\circ} 30'$ revolves rapidly in front of the plate, by means of the band H I passing over the pulley G, so as to project beyond the aperture A, which is only visible when the sector passes before it at each revolution of the disc.* The apertures were first illuminated by gas-light, and the disc being made to revolve so rapidly as to produce a continuous impression, the apparent brightness of the apertures was made equal by turning one of the prisms. When the apparatus was next illuminated by the direct light of the sun at noon, and the disc made to revolve so as to produce a uniform impression, the apertures were still equally bright, although the position of the prisms remained unaltered. This experiment was repeated several times with the same result, and a similar result was obtained when moon-light was compared with gas-light. Now the effect of turning round the prism is to diminish the brightness both of the sun-light and gas-light in the same proportion. Since, therefore, the two apertures were always equally bright, it follows, that the apparent brightness of the aperture behind the revolving disc, had also, in both cases, the same ratio to that of the light seen by uninterrupted vision. But the ratio of the apparent brightness of the aperture behind the revolving disc to that of the direct light, evidently depends on the rapidity with which the light acts on the eye at each passage of the sector before the luminous aperture. Hence it is obvious, that if the sun-light and gas-light required different times to produce like portions of their total effect on the eye, the apparent brightness of the flashes produced by the revolving disc would have different ratios

* By means of this arrangement, the brightness of the impressions produced during the revolution of the disc, can be compared with the light transmitted through the aperture B. Since the intensity of a ray of polarized light when transmitted through a doubly-refracting crystal, varies as the square of the cosine of the inclination of the principal section of the crystal to the plane of polarization of the ray ; by attaching an index to the tube B D, so as to measure the angle through which it has been turned, the intensity of the transmitted light might be estimated, and thus the brightness of the impressions produced by the revolving disc might be determined. (See *Supplément au Traité de la Lumière* de Sir J. F. W. HERSCHEL. Par A. QUETELET, p. 595.)

to that of the uninterrupted light, according as the apparatus was illuminated by sun-light or by gas-light. Therefore, since it has been shewn that this was not the case, it is evident, that the sun-light and gas-light produced similar portions of their complete impressions on the eye with the same rapidity.

It has thus been proved, that, when light acts on the eye for short intervals of time, the rapidity of the development of its impression is independent of its actual brightness; and it seems highly probable that this law extends to the whole time required for the complete production of luminous impressions. For, when it has been found, that lights of very different intensity acting on the eye during $\frac{1}{50}$ th of a second, all produce impressions, having almost exactly $\frac{1}{5}$ th of the absolute brightness of the lights, it seems natural to conclude, that they will also produce their complete effect on the eye in exactly equal times.

I hoped to have been able in this paper to exhibit the results of some experiments upon the intensity of impressions of short duration, repeated by different individuals, in order to ascertain whether the rapidity of the production of visual impressions varies much in different eyes. I have only obtained one comparison of this kind, through the kindness of Mr ALEXANDER WALLACE, of the Royal Observatory, Edinburgh, who observed the impression produced on his eye by a disc with a sector of $7^{\circ} 30'$, revolving 20 times in a second. The following result is the mean of three trials,

$$d_1 = 4.39; d_2 = 42.5; b = 0.01067.$$

The result of my own experiments gives $b = 0.0137$: which agrees very well with Mr WALLACE's observations. I trust to be able to obtain some more comparisons of this kind, in order to ascertain whether the agreement between Mr WALLACE's result and my own is to be regarded as proving that visual impressions in the eyes of different individuals, are propagated with nearly equal rapidity.

VII. *On the time which Light requires to produce a full impression on the Eye.*

I have found, by means of a disc revolving once in a second (see p. 594), that impressions produced by a light acting on the eye for $\frac{1}{10}$ th of a second, have very nearly the same brightness as the light seen by continuous vision; but that when light acts on the eye for a shorter time, its apparent brightness is sensibly diminished. As the brightness of the impression produced by light increases by insensible degrees, until, at length, it attains its full intensity, it is obviously almost impossible, by direct observation, to assign the exact instant when this takes place. The experiments in Sections V. and VI. have proved, that, up to $\frac{1}{16}$ th of a second, the brightness of a luminous impression is strictly proportional to the time during which light has acted on the eye, and also, that the impression produced in $\frac{1}{100}$ th of a second, has almost exactly $\frac{1}{10}$ th of the brightness of a full impres-

sion. If this proportionality between the duration and the apparent brightness of a light be supposed to extend beyond the limits of the experiments, so as to include nearly the whole time required for the production of a complete impression, it would obviously follow that light requires about $\frac{1}{10}$ th of a second to produce its full effect on the eye; and this conclusion, it will be observed, agrees with the result of direct observation.

The following inferences may be derived from the laws of vision which have now been investigated.

1. *Personal Equation in Astronomy.*

It is well known, that different observers assign different times to the occurrence of the same astronomical phenomenon; as, for example, the passage of a star across the meridian wire of a transit instrument. The correction to be applied to reduce the observations, or personal equation, as it is termed, frequently amounts to a considerable quantity.

It might at first be supposed, that this discrepancy between the results of different observers, may be occasioned by light acting on their eyes with unequal degrees of rapidity. But on considering the manner of observing the transit of a star, it will appear that this explanation is insufficient. In order to estimate the exact time at which the star passes one of the wires, the observer endeavours to recollect the position of the star on one side of the wire, at the instant when he heard the clock beat. At the next beat of the clock, the star has passed to the other side of the wire; and the observer then, by the eye, subdivides into equal parts the space between the positions occupied by the star at the successive beats of the clock, and estimates how many of those parts are contained in the interval between the wire and the first position of the star. The magnitude of that interval estimated in this manner, determines the fraction of a second to be added to the time given by the clock. If, then, the discrepancy between the results of different observers is to be regarded as a phenomenon of vision, it must depend upon some cause which displaces the image of the star, and thereby alters its apparent distance from the wire. Now as the star passes across the field of the telescope, its light falls successively upon different parts of the retina, illuminating each portion for a very small space of time; and if light acted on the eye of one observer more rapidly than on that of another, the obvious consequence would be that the image of the star would appear brighter to the person whose retina was most quickly impressed by light. The only other effect which the gradual action of light on the eye seems capable of producing, is to render the advancing edge of the image of the star so faint, owing to the extremely short time during which its light acts on the eye, as to become imperceptible when contrasted with the succeeding parts of the image: for these fall upon points of the retina over

which a portion of the image has already passed, and on which the light has had time to develop a distinct impression. In this manner, it may be conceived, that the breadth of the image will be diminished on the side towards which it moves, while it will be increased on the other side by the persistence of the impression of light on the eye; and, consequently, the image of the star will appear behind its true position. It is obvious, however, that the retardation of the advancing edge of the image cannot exceed the breadth of the extremely minute disc with which a star appears in a good telescope, otherwise it would amount to a total extinction of the light; and, on the other hand, the image cannot be prolonged by the persistence of its impression on the retina, by a greater quantity than its advancing edge is retarded, without becoming perceptibly elongated. Any difference in the amount of retardation due to such causes, in different eyes, must therefore be confined within extremely narrow limits, and seems quite inadequate to account for the personal equation which, in some instances, amounts to a large fraction of a second.*

2. Rays of Light of different Refrangibility act on the Eye with the same rapidity.

In the observations made with a rapidly revolving disc, where each flash lasted only $\frac{1}{18432}$ of a second, not the slightest alteration in the colour of the luminous object was perceptible. The blue part of a gas flame, indeed, became invisible; but this was evidently due to the great reduction of the intensity of the light rendering the blue rays incapable of producing a sensible impression on the eye, already affected by the more luminous rays. From this it follows, that rays of light of different refrangibility act on the eye with equal rapidity. For if we suppose some of the rays which constitute white light to act on the eye more rapidly than others, the effect of shortening the luminous impressions would be quite analogous to that produced by the interposition of some medium, such as red glass, which absorbs the rays unequally; and the eye would be affected with the complementary colour of the deficient rays.

That there is no sensible difference in the rapidity of the action of lights of various colours on the retina, appears also from the fact, that when the eye is suddenly directed to a luminous object, the first impression of its colour remains afterwards unaltered. This could not be always the case if there was any great inequality in the rapidity with which the different rays produce their effect on the eye. If, for example, we suppose the blue rays to act more rapidly than the yellow rays, green objects would, at first sight, appear to have more of a bluish tinge than after the eye had continued to regard them for a short time.

* As my object here is simply to discuss the possibility of explaining the personal equation by the gradual action of light on the retina, I have intentionally refrained from entering upon any explanation of that phenomenon which may be derived from the supposition that time is required for the transmission of impressions from the organs of sensation to the mind.

3. *On the Difference between the Apparent and the Intrinsic Brightness of the Flash produced by Electricity of high tension.*

Professor WHEATSTONE, as was already noticed, has proved that the light of electricity of high tension has a less duration than the millionth part of a second. Now, since it has been shewn that lights of every intensity produce their impressions on the eye in equal times,* and that the brightness of an impression is exactly proportional to its duration; it follows, that if the electric spark could be made to last for the hundredth part of a second, which is 10,000 times its actual duration, its apparent brightness would also be increased 10,000 times. But the results already recorded (see Table, p. 595), shew that the apparent intensity of light lasting for the hundredth part of a second scarcely exceeds a tenth of its real intensity. Hence, if the duration of the electric spark could be prolonged so as to render its light continuous, its apparent brightness would probably be increased about 100,000 times.

From the nature of the experiments on which this conclusion is founded, it is perhaps only strictly applicable to the case where the electric spark is seen by the eye already acted on by light of moderate intensity; for in other cases its apparent brightness is no doubt greatly increased by the contrast with previous darkness;† but however remarkable the conclusion may appear, it seems perfectly consistent with the estimate of the intrinsic brightness of the electric spark, which arises from reflecting on the extremely short space of time in which its powerful impression on the eye is produced.

Dr Faraday observes, that "the beautiful flash of light attending the discharge of common electricity, rivals in brilliancy, if it does not even very much surpass, the light from the discharge of voltaic electricity;"‡ and again he states, that when a battery of 15 jars was discharged through a wet string, "the spark was yellowish, flamy, and having a duration sensibly longer than if the water had not been interposed." Now the effect of discharging the battery through a bad conductor, would be greatly to diminish the tension of the electricity, while it augmented the duration of the spark. If, therefore, the intrinsic brightness of the spark had remained the same as before, the intensity of the impression on the eye should have been increased; but the reverse seems to have been the case. Hence it follows, that the brightness of the electric spark increases with the tension of the electricity. A similar conclusion may obviously be derived from a compa-

* The electric spark is a light whose intensity places it undoubtedly within the limits of the experiments on this point, as its brightness is inferior to that of sun-light. According to Sir JOHN HERSCHEL, the lime-ball light appears only as a black spot on the disc of the sun when held between it and the eye.—(See *Treatise on Astronomy*, LARDNER'S *Cyclopædia*, p. 210. London, 1835.)

I have observed that, in like manner, the spark produced by a strongly-charged Leyden phial, is absolutely invisible when it passes between the eye and the sun's disc.

† See *Light*. *Encyclopædia Metropolitana*, Art 58.

‡ *Experimental Researches in Electricity*, vol. i., sec. 333. Lond. 1839.

parison of the nearly instantaneous electric spark of high tension, with the apparently continuous light of voltaic electricity. For since the latter light, notwithstanding its sensible duration, does not appear brighter than the former, it must obviously be greatly inferior in intrinsic brightness.

It may now be useful to recapitulate the principal results of the experiments described in this paper.

1. When the eye receives a succession of flashes of equal duration from a light of constant intensity, which succeed each other so rapidly as to produce a uniform impression, the intensity of this aggregate impression will also be constant, provided the number of flashes in a given time varies inversely with the duration of each.

2. The brightness of the impression produced by flashes of light of a given intensity, which succeed each other so rapidly as to produce a uniform impression on the eye, is proportional to the number of flashes in a given time.

3. When light of a given intensity acts on the eye for a short space of time, the brightness of the luminous impression on the retina is exactly proportional to the time during which the light continues to act. This law has been proved to be true for impressions lasting from $\frac{1}{18432}$ to $\frac{1}{24}$ of a second.

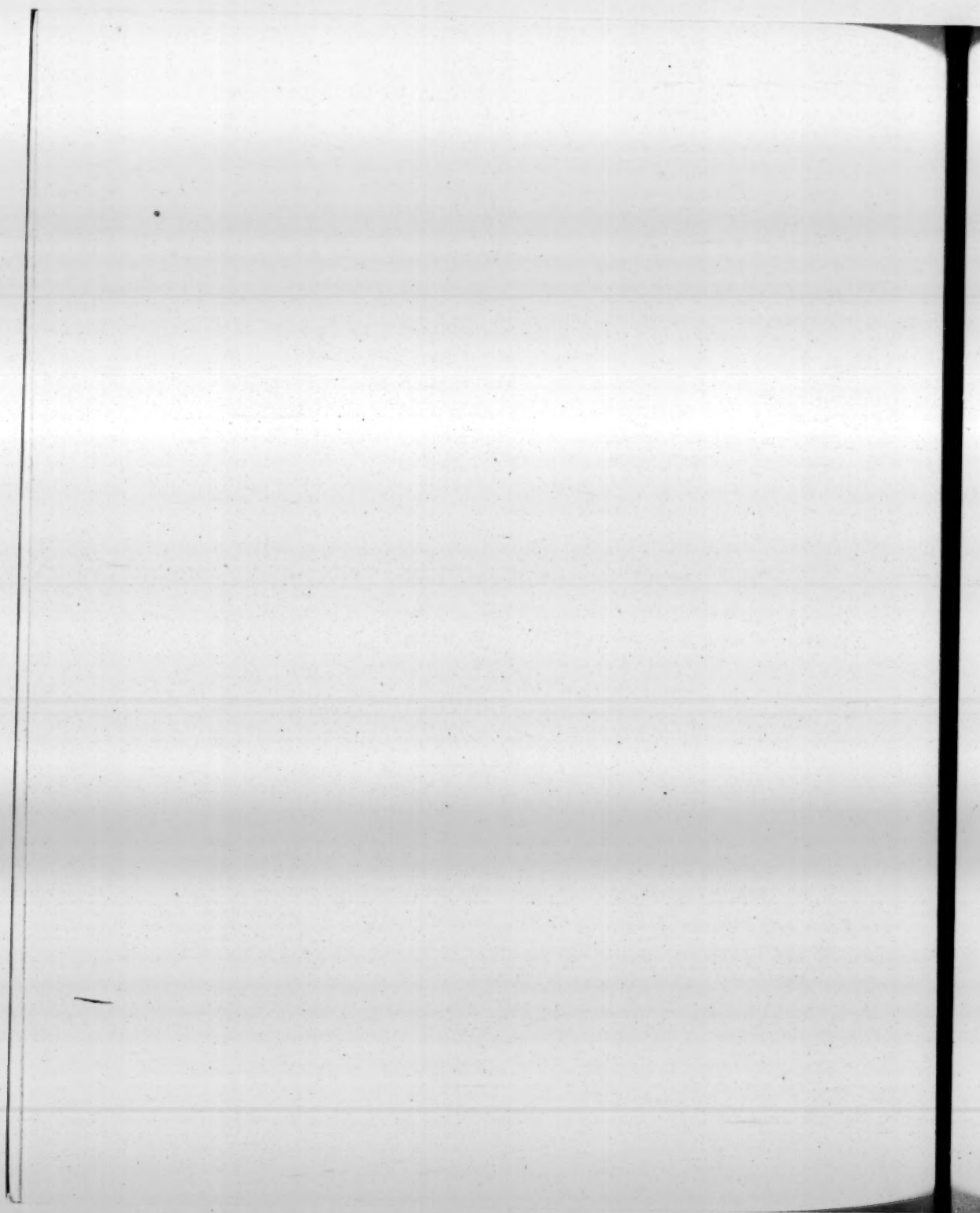
4. The intensity of the impression produced by light which acts on the eye for $\frac{1}{100}$ of a second is almost exactly $\frac{1}{10}$ th of the apparent brightness of the light when seen by uninterrupted vision; and the time required for light to produce its full effect on the eye seems to be about $\frac{1}{10}$ th of a second.

5. Lights of different intensities produce their complete impressions on the eye in equal times, so that the light of the sun requires the same time as common artificial light to produce its impression on the eye.

6. The brightness of an impression on the eye increases with a rapidity exactly proportional to that of the light by which it is produced.

7. Rays of different refrangibility act on the eye with equal rapidity.

8. The apparent brightness of the spark produced by electricity of high tension is only about $\frac{1}{100000}$ th of what its apparent brightness would become if its duration were prolonged to $\frac{1}{10}$ th of a second; and the brightness of electric light increases with the tension of the electricity.



PROCEEDINGS

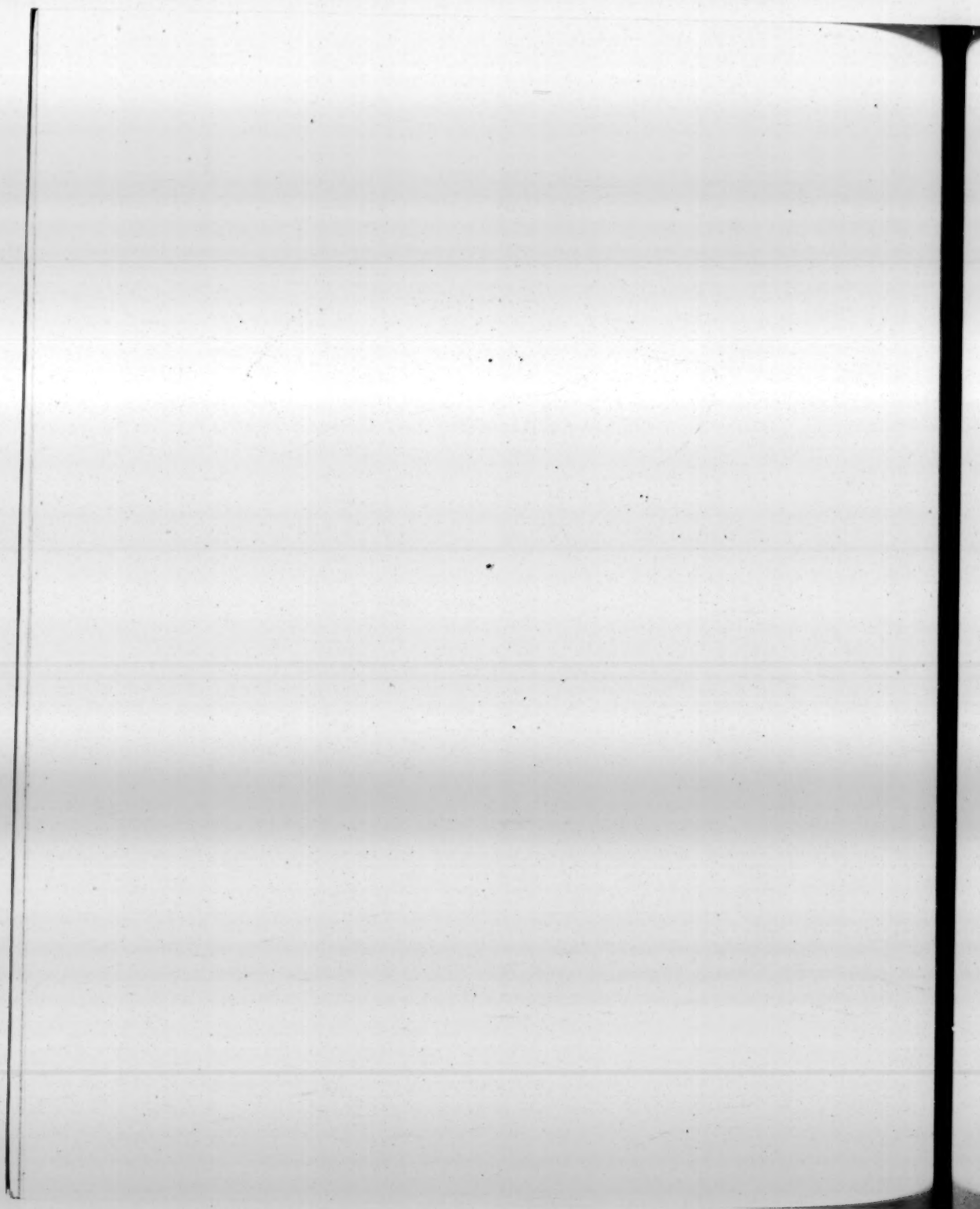
OF THE

EXTRAORDINARY GENERAL MEETINGS,

AND

LISTS OF MEMBERS ELECTED AT THE ORDINARY MEETINGS,

SINCE NOVEMBER 25, 1844.



PROCEEDINGS. &c.

Monday, November 25, 1844.

At a Statutory General Meeting, held for the purpose of appointing Office-Bearers for the ensuing Session, The Right Honourable Earl CATHCART, Vice-President, in the Chair, the Ballot was taken in the usual way, and the following Gentlemen were declared to be duly elected, viz. :—

Sir T. MAKDOUGALL BRISBANE, Bart., President.

Sir WILLIAM MILLER, Bart.,

Sir DAVID BREWSTER, K.H.,

Earl CATHCART,

Very Reverend Principal LEE,

Sir GEORGE S. MACKENZIE, Bart.,

Right Reverend Bishop TERROT,

Professor FORBES, General Secretary.

DAVID MILNE, Esq.,

Dr GREGORY,

JOHN RUSSELL, Esq., Treasurer.

Dr TRAILL, Curator of Library.

JOHN STARK, Esq., Curator of Museum.

Vice-Presidents.

Ordinary Secretaries.

COUNSELLORS.

Dr PARNELL.

Dr CARSON.

Sir JOHN McNEILL.

Sir THOMAS D. LAUDER, Bart.

ALAN STEVENSON, Esq.

JAMES T. GIBSON-CRAIG, Esq.

Dr CRAIGIE.

Professor MILLER.

Professor PILLANS.

Professor KELLAND.

Dr CHRISTISON.

Dr NEILL.

Dr CHRISTISON, as acting General Secretary, read the following Resolution from the Minutes of the Council :—“18th November 1844.—The Council unanimously resolved, That it was expedient to allow a Salary of £100 annually to the General Secretary, on the understanding that the duties of the Office should, henceforth, include the charge of publishing the Society's Proceedings, and that the proposed arrangement should be considered as an experimental

one, which might be altered in the event of its not being found to answer the Council's expectations, or of the Society's funds proving inadequate." Dr CHRISTISON having explained the views which had led the Council to this opinion, Lord MURRAY moved, seconded by JAMES L'AMY, Esq., That the Resolution of the Council be adopted by the Society; which motion was agreed to by the meeting, with one dissident.

On the motion of the Treasurer, the Council-Committee on the Funds were appointed to audit his accounts.

Mr RUSSELL moved, seconded by Dr GREVILLE, That the Royal Society, in consideration of the able and efficient manner in which Dr CHRISTISON, in the absence of Professor FORBES, has, during the last twelvemonth, discharged the duties of General Secretary to the Society, hereby tender their warmest thanks to him for these and all his other valuable services. This motion being carried unanimously, Earl CATHCART conveyed the thanks of the Society to Dr CHRISTISON.

(Signed) C. H. TERROT, V.P.

Monday, November 24, 1845.

At a Statutory General Meeting, Right Reverend Bishop TERROT in the Chair, the following Office-Bearers were duly elected:—

Sir T. M. BRISBANE, Bart., G.C.B., President.

Sir WILLIAM MILLER, Bart.,

Sir D. BREWSTER, K.H.,

Very Reverend Principal LEE,

Sir G. S. MACKENZIE, Bart.,

Right Reverend Bishop TERROT,

Dr CHRISTISON,

Professor FORBES, General Secretary.

D. MILNE, Esq.,

Dr GREGORY.

JOHN RUSSELL, Esq., Treasurer.

Dr TRAILL, Curator of Library.

JOHN STARK, Esq., Curator of Museum.

Vice-Presidents.

Secretaries to Ordinary Meetings.

COUNSELLORS.

ALAN STEVENSON, Esq.

J. T. GIBSON-CRAIG, Esq.

Dr CRAIGIE.

Professor MILLER.

Professor KELLAND.

Professor PILLANS.

Dr NEILL.

Dr FLEMING.

Mr ADIE.

Lord MURRAY.

Dr BRUNTON.

G. FORBES, Esq.

The following Committee was appointed to audit the Treasurer's accounts:—

Sir H. JARDINE.

JAMES GIBSON-CRAIG, Esq.

GEORGE FORBES, Esq.

Part of the Minute of Council of the 19th November having been read. relative to the Salary of the General Secretary, it was moved by Dr MACLAGAN, "That the Royal Society having considered the recommendation of the Council of the 19th November, that the Salary of £100 should be continued to Professor FORBES, as General Secretary, unanimously approve of the said recommendation, and resolve that the said Salary of £100 shall be continued to Mr FORBES;" which motion having been seconded by Mr CADELL, was unanimously agreed to.

It was farther moved and seconded, and unanimously agreed to, That, in conformity to a recommendation by the Council, the name of JAMES SKENE, Esq. of Rubislaw, now returned from the Continent, shall (if agreeable to him) be replaced in the list of Fellows of the Society.

(Signed) G. S. MACKENZIE, V.P.

Monday, November 23, 1846.

At a Statutory General Meeting, Sir G. S. MACKENZIE, Bart., V.P., in the Chair, the following Office-Bearers were duly elected:—

Sir T. MAKDOUGALL BRISBANE, Bart., G.C.B., G.C.H., President.

Sir D. BREWSTER, K.H.,

Right Hon. Earl CATHCART,

Very Rev. Principal LEE,

Sir GEORGE S. MACKENZIE, Bart.,

The Right Rev. Bishop TERROT,

Dr CHRISTISON,

Professor FORBES, General Secretary.

DAVID MILNE, Esq.,

Dr GREGORY,

JOHN RUSSELL, Esq., Treasurer.

Dr TRAILL, Curator of Library and Instruments.

JOHN STARK, Esq., Curator of Museum.

Vice-Presidents.

Secretaries to the Ordinary Meetings.

COUNSELLORS.

Professor KELLAND.

Professor PILLANS.

Dr NEILL.

Rev. Dr FLEMING.

A. ADIE, Esq.

Hon. Lord MURRAY.

Rev. Dr BRUNTON.

GEORGE FORBES, Esq.

W. A. CADELL, Esq.

Sir WM. SCOTT, Bart.

Dr J. H. BALFOUR.

HENRY MARSHALL, Esq.

On the motion of Sir G. S. MACKENZIE, it was resolved, unanimously, That the name of Earl CATHCART be replaced according to his former standing in the List of Vice-Presidents.

The following gentlemen were named a Committee to audit the Treasurer's Accounts:—

GEORGE FORBES, Esq.

D. SMITH, Esq.

J. T. GIBSON-CRAIG, Esq.

The Meeting then adjourned.

(Signed) C. H. TERROT, V.P.

Monday, November 22, 1847.

At a Statutory General Meeting, Bishop TERROT in the Chair, the following Office-Bearers for the ensuing Session were duly elected :—

Sir T. MAKDOUGALL BRISBANE, Bart., G.C.B., G.C.H., President.		
Sir D. BREWSTER, K.H.,	} Vice-Presidents.	
Right Hon. Earl CATHCART,		
Very Rev. Principal LEE,		
Sir GEORGE S. MACKENZIE, Bart.,		
Right Rev. Bishop TERROT,		
Dr CHRISTISON,	} Secretaries to the Ordinary Meetings.	
Professor FORBES, General Secretary.		
DAVID MILNE, Esq.,		
Dr GREGORY,		
JOHN RUSSELL, Esq., Treasurer.		
Dr TRAILL, Curator of Library and Instruments.		
JOHN STARK, Esq., Curator of Museum.		

COUNSELLORS.

A. ADIE, Esq.	Dr J. H. BALFOUR.
HON. LORD MURRAY.	HENRY MARSHALL, Esq.
Rev. Dr BRUNTON.	Sir WM. JARDINE, Bart.
GEORGE FORBES, Esq.	Prof. C. PIAZZI SMYTH.
W. A. CADELL, Esq.	Rev. Dr ROBERTSON.
Sir WM. SCOTT, Bart.	C. MACLAREN, Esq.

The following gentlemen were appointed to audit the Treasurer's Accounts :—

GEORGE FORBES, Esq. DAVID SMITH, Esq. JAMES T. GIBSON-CRAIG, Esq.

The Meeting then adjourned.

(Signed) C. H. TERROT, V.P.

Memorandum.—February 21, 1848.—At an Ordinary Meeting of the Royal Society, on the 21st February, the following motion for a change of Law, was proposed by Mr RUSSELL, Treasurer, the motion itself having been announced from the Chair on the 17th January, and printed in the Billets of the 7th and 21st February, viz :—

“That Law IV. shall be altered, and stand as follows :—

“IV. The Fees of Admission of an Ordinary Non-Resident Fellow shall be £26, 5s., payable on his admission ; and in case of any Non-Resident Fellow coming to reside at any time in Scotland, he shall, during each year of his residence, pay the usual Annual Contribution of £3, 3s. payable by each Resident Fellow ; but after payment of such Annual Contribution for eight years, he shall be exempt from any farther payment.”

The motion was seconded by Dr TRAILL, and was adopted.

(Signed) C. H. TERROT, V.P.

Monday, November 27, 1848.

At a Statutory General Meeting, Right Rev. Bishop TERROT V.P., in the Chair, the following Office-Bearers for the ensuing year, were duly elected :—

Sir T. MAKDOUGALL BRISBANE, Bart., G.C.B., G.C.H., President.	
Sir D. BREWSTER, K.H.,	}
Right Hon. Earl CATHCART,	
Very Rev. Principal LEE,	
Right Rev. Bishop TERROT,	
Dr CHRISTISON,	
Dr ALISON,	
Professor FORBES, General Secretary.	}
Dr GREGORY,	
Professor C. P. SMYTH,	
JOHN RUSSELL, Esq., Treasurer.	}
Dr TRAILL, Curator of Library and Instruments.	
JOHN STARK, Esq., Curator of Museum.	

Vice-Presidents.

Secretaries to the Ordinary Meetings.

COUNSELLORS.

W. A. CADELL, Esq.	C. MACLAREN, Esq.
Sir Wm. SCOTT, Bart.	D. MILNE, Esq.
Dr J. H. BALFOUR.	J. T. GIBSON-CRAIG, Esq.
HENRY MARSHALL, Esq.	Dr GEORGE WILSON.
Sir Wm. JARDINE, Bart.	Sir JOHN MACNEILL, G.C.B.
Rev. Dr ROBERTSON.	JAMES DALMAHOY, Esq.

The following Committee was named to audit the Treasurer's Accounts.

Dr NEILL. JAMES T. GIBSON-CRAIG, Esq. JAMES WILSON, Esq.

The Meeting then adjourned.

MEMBERS ELECTED.

January 6, 1845.

JAMES ANDREW, M.D. GEORGE WILSON, M.D.

February 3, 1845.

JOHN G. M. BURT, M.D. THOMAS ANDERSON, M.D.

January 5, 1846.

A. TAYLOR, M.D., Pau. S. A. PAGAN, M.D.

February 2, 1846.

Rev. Dr JAMES ROBERTSON. ALEXANDER J. ADIE, Esq.

PROCEEDINGS OF GENERAL MEETINGS,

February 16, 1846.

WILLIAM MURRAY, Esq., of Henderland.

March 16, 1846.

GEORGE TURNBULL, Esq.

GEORGE J. GORDON, Esq.

Dr L. SCHMITZ, Rector of High School.

December 7, 1846.

CHARLES PIAZZI SMYTH, Esq., Professor of Astronomy.

January 4, 1847.

GEORGE MARGILL, Esq.

DAVID GRAY, Esq., Professor of Nat. Philosophy, Marischal College, Aberdeen.

February 1, 1847.

WILLIAM THOMSON, Esq., Prof. of Nat. Philosophy, Glasgow.

J. H. BURTON, Esq., Advocate.

March 1, 1847.

JAMES NICOL, Esq., London.

April 19, 1847.

W. MACDONALD MACDONALD, Esq., of St Martins.

ROBERT HANDYSIDE, Esq., Advocate.

ALEXANDER CHRISTIE, Esq.

December 20, 1847.

JOHN WILSON, Esq., Cirencester.

MOSES STEVEN, Esq., of Bellahouston.

January 17, 1848.

JAMES TOD, Esq., W.S.

March 6, 1848.

THOMAS STEVENSON, Esq., C.E.

JAMES ALLAN, M.D., Haslar Hospital.

JOHN HALL MAXWELL, Esq., Younger of Dargavel.

March 20, 1848.

REV. JOHN HANNAH.

HENRY DAVIDSON, Esq.

April 17, 1848.

PATRICK NEWBIGGING, M.D.

WILLIAM SWAN, Esq.

December 4, 1848.

REV. FRANCIS GARDEN.

December 18, 1848.

PATRICK JAMES STIRLING, Esq.

January 2, 1849.

WILLIAM STIRLING, Esq., of Keir.

D. R. HAY, Esq.

JOHN THOMSON GORDON, Esq., Sheriff of Mid-Lothian.

WILLIAM THOMAS THOMSON, Esq.

Rt. Hon. ANDREW RUTHERFURD, Lord Advocate for Scotland.

Honourable LORD IVORY.

February 5, 1849.

ADAM ANDERSON, Esq., Advocate.

February 19, 1849.

WILLIAM E. ATTOUN, Esq., Professor of Rhetoric and Belles Lettres, University, Edinburgh.

W. H. LOWE, M.D.

March 19, 1849.

Hon. B. F. PRIMROSE.

JOHN STENHOUSE, M.D., Glasgow.

DAVID ANDERSON, Esq., of Moredun.

April 2, 1849.

W. R. PIRRIE, M.D., Professor of Surgery, Marischal College, Aberdeen.

Right Hon. The EARL of MINTO.

April 16, 1849.

Right Hon. The EARL of ABERDEEN.

Right Hon. The EARL of HADDINGTON.

LIST OF THE PRESENT ORDINARY MEMBERS,

IN THE ORDER OF THEIR ELECTION.

Major-General Sir THOMAS M. BRISBANE, Bart., G.C.B., &c., F.R.S. Lond.,
PRESIDENT.

Date of
Election.

- 1798 Alexander Monro, M.D.
1799 Robert Jameson, Esq., *Professor of Natural History.*
1805 Thomas Thomson, M.D., F.R.S. Lond., *Professor of Chemistry, Glasgow.*
George Dunbar, Esq., *Professor of Greek.*
1807 John Campbell, Esq., *of Carbrook.*
Thomas Thomson, Esq., *Advocate.*
1808 James Wardrop, Esq.
Sir David Brewster, K.H., LL.D., F.R.S. Lond., *St Andrews.*
1811 Major-General Sir Thomas Makdougall Brisbane, Bart., G.C.B., G.C.H., F.R.S. Lond.
James Jardine, Esq., *Civil Engineer.*
J. G. Children, Esq., F.R.S. Lond.
Alexander Gillespie, Esq., *Surgeon.*
W. A. Cadell, Esq., F.R.S. Lond.
James Pillans, Esq., *Professor of Humanity.*
1812 Sir George Clerk, Bart., F.R.S. Lond.
1813 William Somerville, M.D., F.R.S. Lond.
1814 Sir Henry Jardine.
Patrick Neill, LL.D., *Secretary to the Wernerian and Horticultural Societies.*
Right Honourable Lord Viscount Arbuthnot.
John Fleming, D.D., *Professor of Natural Science, New College.*
Alexander Brunton, D.D.
1815 Robert Stevenson, Esq., *Civil Engineer.*
Henry Home Drummond, Esq., *of Blair-Drummond.*
William Thomas Brande, Esq., F.R.S. Lond., *Professor of Chemistry in the Royal
Institution.*

Date of
Election.

- 1816 Leonard Horner, Esq., F.R.S. Lond.
Henry Colebrooke, Esq., *Director of the Asiatic Society of Great Britain.*
Honourable Lord Fullerton.
- 1817 Right Honourable Earl of Wemyss and March.
John Wilson, Esq., *Professor of Moral Philosophy.*
Alexander Maconochie, Esq., *of Meadowbank.*
Sir David James Hamilton Dickson, M.D., *Clifton.*
William P. Alison, M.D., *Professor of the Practice of Physic.*
Robert Bald, Esq., *Civil Engineer.*
- 1818 Robert Richardson, M.D., *Harrougate.*
Patrick Miller, M.D., *Exeter.*
John Watson, M.D.
Right Honourable John Hope, *Lord Justice-Clerk.*
- 1819 Sir Patrick Murray, *of Simprim.*
James Muttiebury, M.D., *Bath.*
Thomas Stewart Traill, M.D., *Professor of Medical Jurisprudence.*
Alexander Adie, Esq.
Marshall Hall, M.D., *London.*
Richard Philips, Esq., F.R.S. Lond.
Reverend William Scoresby, *Exeter.*
George Forbes, Esq.
- 1820 Right Honourable David Boyle, *Lord Justice-General.*
James Keith, Esq., *Surgeon.*
Charles Babbage, Esq., F.R.S. Lond.
Thomas Guthrie Wright, Esq., *Auditor of the Court of Session.*
Sir John F. W. Herschel, Bart., F.R.S. Lond.
John Shank More, Esq., *Professor of Scots Law.*
Robert Haldane, D.D., *Principal of St Mary's College, St Andrews.*
Sir John Mead, M.D., *Weymouth.*
Dr William Macdonald.
Sir John Hall, Bart., *of Dunglass.*
Sir George Ballingall, M.D., *Professor of Military Surgery.*
- 1821 Sir James M. Riddell, Bart., *of Ardnamurchan.*
Archibald Bell, Esq., *Advocate.*
John Clerk Maxwell, Esq., *Advocate.*
John Lizars, Esq., *Surgeon.*
John Cay, Esq., *Advocate.*
Robert Kaye Greville, LL.D.
Robert Hamilton, M.D.
A. R. Carson, Esq., LL.D.
- 1822 James Smith, Esq., *of Jordankill,* F.R.S. Lond.
William Bonar, Esq.
George A. Walker-Arnott, Esq., LL.D., *Professor of Botany, Glasgow.*
Very Reverend John Lee, D.D., *Principal of the University.*

Date of
Election.

- 1822 Sir James South, F.R.S. Lond.
 Lieutenant-General Martin White.
 Walter Frederick Campbell, Esq. of *Shawfield*, M.P.
 Sir W. C. Trevelyan, Bart., *Nettlecombe, Somersetshire*.
 Sir Robert Abercromby, Bart., of *Birkenbog*.
 Dr Wallich, *Calcutta*.
 John Russell, Esq., P.C.S.
 John Dewar, Esq., *Advocate*.
- 1823 Sir Edward Ffrench Bromhead, Bart., A.M., F.R.S. Lond. *Thurlesby Hall*.
 Captain Thomas David Stuart, of the *Hon. East India Company's Service*.
 Andrew Fyfe, M.D., *Professor of Medicine and Chemistry, King's College, Aberdeen*.
 Robert Bell, Esq., *Advocate*.
 Captain Norwich Duff, R.N.
 Warren Hastings Anderson, Esq.
 Alexander Thomson, Esq. of *Banchory*.
 Liscombe John Curtis, Esq., *Ingsdon House, Devonshire*.
 Robert Christison, M.D., *Professor of Materia Medica*.
 John Gordon, Esq., of *Cairnbulg*.
- 1824 Dr Lawson Whalley, *Lancaster*.
 Alexander Wilson Philip, M.D., *London*.
 Sir Charles Adam, R.N.
 Robert E. Grant, M.D., *Professor of Comparative Anatomy, Univ. Coll., London*.
 Rev. Dr William Muir, one of the *Ministers of Edinburgh*.
 W. H. Playfair, Esq., *Architect*.
 John Argyle Robertson, Esq., *Surgeon*.
 James Pillans, Esq.
 James Walker, Esq., *Civil Engineer*.
 Sir William Newbigging, *Surgeon*.
 William Wood, Esq., *Surgeon*.
- 1825 The Venerable Archdeacon John Williams.
 W. Preston Lauder, M.D., *London*.
 Right Honourable Lord Ruthven.
 Sir William Jardine, Bart., of *Applegarth*.
 Honourable Lord Wood.
- 1826 Sir David Hunter Blair, Bart.
 John Stark, Esq.
 Dr John Macwhirter.
- 1827 John Gardiner Kinnear, Esq.
 James Russell, M.D.
 Rev. Dr Robert Gordon, one of the *Ministers of Edinburgh*.
 James Wilson, Esq.
 Very Rev. Edward Bannerman Ramsay, A.M., *Camb*.
 George Swinton, Esq.
- 1828 Erskine Douglas Sandford, Esq., *Advocate*.

LIST OF ORDINARY MEMBERS.

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Date of
Election.

- 1828 David MacLagan, M.D.
Sir William Maxwell, Bart.
John Forster, Esq., *Architect, Liverpool.*
Thomas Graham, A.M., *Professor of Chemistry, London University.*
David Milne, Esq., *Advocate.*
Dr Manson, *Nottingham.*
William Burn Callender, Esq., *of Prestonhall.*
- 1829 A. Colyar, Esq.
William Gibson-Craig, Esq., M.P.
James Ewing, LL.D., *Glasgow.*
Duncan McNeill, Esq., *Dean of Faculty.*
Ven. Archdeacon Sinclair, *Kensington.*
Arthur Connell, Esq., *Professor of Chemistry, St Andrews.*
Bindon Blood, Esq., M.R.I.A.
James Walker, Esq., W.S.
William Bald, Esq., M.R.I.A.
- 1830 J. T. Gibson-Craig, Esq., W.S.
Archibald Alison, Esq., *Sheriff of Lanarkshire.*
Honourable Mountstuart Elphinstone.
James Syme, Esq., *Professor of Clinical Surgery.*
Thomas Brown, Esq., *of Lanfine.*
James L'Amy, Esq., *Sheriff of Forfarshire.*
Thomas Barnes, M.D., *Carlisle.*
- 1831 James D. Forbes, Esq., F.R.S. Lond., *Professor of Natural Philosophy.*
Right Honourable Lord Dunfermline.
Donald Smith, Esq.
Captain Sir Samuel Brown, R.N.
O. Tyndal Bruce, Esq., *of Falkland.*
David Boswell Reid, M.D., *London.*
T. S. Davies, Esq., A.M., *Woolwich.*
- 1832 John Sligo, Esq. *of Carmyle.*
James Dunlop, Esq., *New South Wales.*
James F. W. Johnston, A.M., *Professor of Chemistry in the University of Durham.*
William Gregory, M.D., *Professor of Chemistry.*
Robert Allan, Esq., *Advocate.*
Robert Morrieson, Esq., *Hon. E.I.C. Civil Service.*
Montgomery Robertson, M.D.
- 1833 Captain Milne, R.N.
His Grace the Duke of Buccleuch.
A. T. J. Gwynne, Esq.
David Craigie, M.D.
George Buchanan, Esq., *Civil Engineer.*
Sir John Stuart Forbes, Bart., *of Pitsligo.*
Alexander Hamilton, Esq., LL.B., W.S.

Date of
Election.

- 1833 Right Honourable Earl Cathcart.
- 1834 Mungo Ponton, Esq., W.S.
Isaac Wilson, M.D., F.R.S. Lond.
David Low, Esq., *Professor of Agriculture.*
Patrick Boyle Mure Macredie, Esq., *Advocate.*
John Davie Morries Stirling, Esq.
Thomas Jameson Torrie, Esq.
William Copland, Esq., *of Colliston.*
John Steuart Newbigging, Esq., W.S.
John Haldane, Esq., *Haddington.*
- 1835 John Hutton Balfour, M.D., *Professor of Botany.*
William Sharpey M.D., *Professor of Anatomy, University College, London.*
Right Honourable Lord Campbell.
William Brown, Esq., F.R.C.S.
Reverend Edward Craig.
R. Mayne, Esq.
- 1836 David Rhind, Esq., *Architect.*
Archibald Robertson, M.D., F.R.S. Lond.
Sir J. Macpherson Grant, *of Ballindalloch.*
Alexander Gibson Carmichael, Esq.
- 1837 John Archibald Campbell, Esq., W.S.
John Scott Russell, Esq., A.M.
Charles Maclaren, Esq.
A. Smith, Esq., M.A. Camb., *Lincoln's Inn, London,*
Richard Parnell, M.D.
Peter D. Handyside, M.D., F.R.C.S.
- 1838 William Nicol, Esq.
William Scott, Esq., *H.E.I.C. Service.*
Thomas Mansfield, Esq., *Accountant.*
Alan Stevenson, Esq., *Civil Engineer.*
- 1839 James Auchinleck Cheyne, Esq., *of Kilmaron.*
David Smith, Esq., W.S.
Adam Hunter, M.D.
Rev. Philip Kelland, A.M., *Professor of Mathematics.*
Henry Marshall, Esq., *Dep. Inspector-General of Army Hospitals.*
William Alexander, Esq., W.S.
F. Brown Douglas, Esq., *Advocate.*
Lieutenant-Colonel Swinburne.
- 1840 Alan A. Maconochie, Esq., *Professor of Civil Law, Glasgow.*
Martyn J. Roberts, Esq.
Robert Chambers, Esq.
James Forsyth, Esq.
Sir John M'Neill, G.C.B.
John Cockburn, Esq.

Date of
Election.

- 1840 Sir William Scott, Bart., *of Ancrum*.
 Right Rev. Bishop Terrot.
 Robert Bryson, Esq.
 Edward J. Jackson, Esq.
 John Learmonth, Esq., *of Dean*.
 John Mackenzie, Esq.
 John Anstruther, Esq., W.S.
- 1841 Colonel Morison, C.B., *Madras Artillery*.
 John Miller, Esq., *Civil Engineer*.
 George Smyttan, M.D.
 James Dalmahoy, Esq.
- 1842 James Thomson, Esq., *Civil Engineer, London*.
 John Davy, M.D., *Inspector-General of Hospitals*.
 Robert Nasmyth, Esq., F.R.C.S.
 Sir James Forrest, Bart., *of Comiston*.
 James Miller, Esq., *Professor of Surgery*.
 John Adie, Esq.
 John Goodsir, Esq., *Professor of Anatomy*.
- 1843 A. D. MacLagan, M.D., F.R.C.S.
 John Rose Cormack, M.D., F.R.C.P., *Putney*.
 Allen Thomson, M.D., *Professor of Anatomy, Glasgow*.
 Joseph Mitchell, Esq., *Civil Engineer, Inverness*.
 Duncan Davidson, Esq., *of Tulloch*.
 Andrew Coventry, Esq., *Advocate*.
 John Hughes Bennett, M.D., F.R.C.P., *Professor of Physiology*.
 D. Balfour, Esq., *Younger of Trenaby*.
 Henry Stephens, Esq.
- 1844 The Honourable Lord Murray.
 Arthur Forbes, Esq., *of Culloden*.
 J. Burn Murdoch, Esq., *Advocate*.
 Archibald Swinton, Esq., *Professor of Civil Law*.
 James Begbie, M.D., F.R.C.S.
 James Y. Simpson, M.D., *Professor of Midwifery*.
 David Stevenson, Esq., *Civil Engineer*.
 Thomas R. Colledge, M.D., F.R.C.P.E.
- 1845 James Andrew, M.D.
 George Wilson, M.D.
 John G. M. Burt, M.D.
 Thomas Anderson, M.D.
- 1846 A. Taylor, M.D., *Pau*.
 S. A. Pagan, M.D.
 Rev. Dr James Robertson, *Professor of Divinity and Ecclesiastical History*.

Date of
Election.

- 1846 Alexander J. Adie, Esq., *Civil Engineer.*
 William Murray, Esq., *of Henderland.*
 George Turnbull, Esq.
 George J. Gordon, Esq.
 Dr L. Schmitz, *Recor of High School.*
 Charles Piazzi Smyth, Esq., *Professor of Practical Astronomy.*
- 1847 George Makgill, Esq., *of Kemback.*
 David Gray, Esq., *Professor of Natural Philosophy, Marischal College, Aberdeen.*
 William Thomson, Esq., M.A., Camb., *Professor of Natural Philosophy, Glasgow.*
 J. H. Burton, Esq., *Advocate.*
 James Nicol, Esq., *Assistant Secretary of the Geological Society, London.*
 W. Macdonald Macdonald, Esq., *of St Martins.*
 Robert Handyside, Esq., *Advocate.*
 Alexander Christie, Esq.
 John Wilson, Esq., *Agricultural College, Cirencester.*
 Moses Steven, Esq., *of Bellahouston.*
- 1848 James Tod, Esq., W.S., *Secretary to the Royal Scottish Society of Arts.*
 Thomas Stevenson, Esq., C.E.
 James Allan, M.D., *Haslar Hospital.*
 John Hall Maxwell, Esq., *Younger of Dargavel.*
 Rev. John Hannah, *Rector of the Edinburgh Academy.*
 Henry Davidson, Esq.
 Patrick Newbigging, M.D.
 William Swan, Esq.
 Rev. Francis Garden.
 Patrick James Stirling, Esq.
- 1849 William Stirling, Esq., *of Keir.*
 John Thomson Gordon, Esq., *Sheriff of Mid-Lothian.*
 Right Honourable Andrew Rutherford, *Lord Advocate for Scotland.*
 D. R. Hay, Esq.
 William Thomas Thomson, Esq.
 Honourable Lord Ivory.
 Adam Anderson, Esq., *Advocate.*
 William E. Aytoun, Esq., *Professor of Rhetoric and Belles Lettres.*
 W. H. Lowe, M.D., *Balgreen.*
 Honourable B. F. Primrose.
 John Stenhouse, M.D., *Glasgow.*
 David Anderson, Esq., *of Moredun.*
 W. R. Pirrie, M.D., *Professor of Surgery, Marischal College, Aberdeen.*
 Right Honourable The Earl of Minto.
 Right Honourable The Earl of Aberdeen.
 Right Honourable The Earl of Haddington.

LIST OF NON-RESIDENT AND FOREIGN MEMBERS.

ELECTED UNDER THE OLD LAWS.

NON-RESIDENT.

Sir James Macgrigor, Bart., M.D.
 Richard Griffiths, Esq., *Civil-Engineer.*

FOREIGN.

Dr S. L. Mitchell, *New York.*
 M. P. Prevost, *Geneva.*

LIST OF HONORARY FELLOWS.

His Majesty the King of the Belgians.
 His Imperial Highness the Archduke John of Austria.
 His Royal Highness the Archduke Maximilian.
 His Royal Highness Prince Albert.

BRITISH SUBJECTS (LIMITED TO TWENTY, BY LAW X.)

J. C. Adams, Esq.,	<i>St John's College, Cambridge.</i>
G. B. Airy, Esq.,	<i>Greenwich.</i>
Robert Brown, Esq.,	<i>London.</i>
Sir M. I. Brunel,	<i>Do.</i>
Dr Faraday,	<i>Do.</i>
Sir John Franklin,	<i>Do.</i>
Professor Graham,	<i>Do.</i>
Henry Hallam, Esq.,	<i>Do.</i>
Sir W. R. Hamilton,	<i>Dublin.</i>
Sir John F. W. Herschel, Bart.,	<i>Collingwood.</i>
Sir William J. Hooker,	<i>Kew.</i>
William Lassell, Esq.,	<i>Starfield, Liverpool.</i>
Dr Lloyd,	<i>Dublin.</i>
Sir Charles Lyell,	<i>London.</i>
Sir Roderick I. Murchison,	<i>Do.</i>
Richard Owen, Esq.,	<i>Do.</i>
Sir W. E. Parry,	<i>Do.</i>
Earl of Rosse, <i>Pres. R.S. Lond.,</i>	<i>Parsonstown.</i>
Rev. Dr Whewell,	<i>Cambridge.</i>
William Wordsworth, Esq.,	<i>Rydal.</i>

LIST OF HONORARY FELLOWS.

FOREIGNERS (LIMITED TO THIRTY-SIX.)

M. Arago,	<i>Paris.</i>
M. Biot,	<i>Do.</i>
M. de Hammer,	<i>Vienna.</i>
M. de Humboldt,	<i>Berlin.</i>
M. Gay-Lussac,	<i>Paris.</i>
M. Agassiz,	<i>Neufchatel.</i>
M. Audubon,	<i>United States.</i>
Sir Henry Bernstein,	<i>Berlin.</i>
M. de Buch,	<i>Do.</i>
M. Cauchy,	<i>Paris.</i>
M. de Charpentier,	<i>Bex.</i>
M. Cousin,	<i>Paris.</i>
M. Degerando,	<i>Do.</i>
M. Charles Dupin,	<i>Do.</i>
M. Ehrenberg,	<i>Berlin.</i>
M. Elie de Beaumont,	<i>Paris.</i>
M. Encke,	<i>Berlin.</i>
M. Flourens,	<i>Paris.</i>
M. Gauss,	<i>Göttingen.</i>
M. Guizot,	<i>Paris.</i>
M. Haidinger,	<i>Vienna.</i>
M. Hansteen,	<i>Christiania.</i>
M. Hausmann,	<i>Göttingen.</i>
M. Jacobi,	<i>Königsberg.</i>
M. Lamont,	<i>Munich.</i>
M. Leverrier,	<i>Paris.</i>
M. Liebig,	<i>Giessen.</i>
M. Melloni,	<i>Naples.</i>
M. Mitscherlich,	<i>Berlin.</i>
M. Müller,	<i>Do.</i>
M. Neander,	<i>Do.</i>
M. Necker,	<i>Geneva.</i>
M. Oersted,	<i>Copenhagen.</i>
M. Plana,	<i>Turin.</i>
M. Quételet,	<i>Brussels.</i>
M. Gustav Rose,	<i>Berlin.</i>
M. Schumacher,	<i>Altona.</i>
M. Struve,	<i>Pulkowa.</i>
M. Thenard,	<i>Paris.</i>
M. Tiedemann,	<i>Heidelberg.</i>

LIST OF FELLOWS DECEASED, RESIGNED, AND CANCELLED,
FROM JULY 1844 TO 1849.

HONORARY FELLOWS.

M. Bessel, *Königsberg*.
M. Alexandre Brongniart, *Paris*.
M. le Baron Berzelius, *Stockholm*.

ORDINARY FELLOWS DECEASED OR RESIGNED.

John Borthwick, Esq., *of Crookston*.
Dr George Cook.
Professor George Glennie.
Dr Robert Graham.
Professor Thomas Henderson.
Dr James Home.
James Hunter, Esq., *of Thurston*.
A. Kinnear, Esq.
Colonel Robertson Macdonald.
Sir D. Milne.
Rev. Dr Welsh.
Henry T. Witham, Esq.
Rev. Dr Bennie.
Dr John Clark.
William Rennie, Esq., W.S.
Claud Russell, Esq.
Dr John Thomson.
Sir William Miller, Bart., *of Glenlee*.
Hugh Murray, Esq.
Sir Archibald Campbell, Bart.
Dr A. Anderson.
His Grace The Duke of Argyll.
Captain D. Boswall, R.N.
Rev. Dr Chalmers.
Dr Davidson.
Sir G. Macpherson Grant.
Professor Macvey Napier.
James Nairne, Esq., W.S.
Rev. Dr Traill.
Sir Thomas Dick Lauder, Bart.
Sir George Stuart Mackenzie, Bart.
Sir Charles G. Menteath, Bart.
James Kinnear, Esq., W.S.

LIST OF MEMBERS RESIGNED AND ELECTIONS CANCELLED.

RESIGNATIONS.

William Burn, Esq.
Nicholas Grut, Esq.
A. Earle Monteith, Esq., *Advocate*.
James Stark, M.D.
Lieutenant-Colonel Low.
Robert Paul, Esq.
William Fergusson, Esq.
J. P. Muirhead, Esq., *Advocate*.

ELECTIONS CANCELLED.

J. Anderson, Esq., C. E.
W. Bell, Esq.
Dr Couper.
Dr W. Ferguson.
William Paul, Esq.
Dr Reid Clanny.
James Hamilton, Esq.
Dr Knox.
W. H. Norie, Esq.

LIST OF DONATIONS.

(Continued from Vol. XV., p. 722.)

December 2, 1844.

DONATIONS.	DONORS.
Transactions of the American Philosophical Society, held at Philadelphia, for Promoting Useful Knowledge. Vol. ix., Part 1.	The Society.
Bulletin de la Société Géologique de France. Tome xiii.	The Society.
Memoirs and Proceedings of the Chemical Society. Parts 7, 8, 9.	Ditto.
Journal of the Statistical Society of London. Vol. vii., Parts 1, 2, 3.	Ditto.
Journal of the Asiatic Society of Bengal, 1843. No. 142.	Ditto.
Det Kongelige Danske Videnskabernes Selskabs Naturvidenskabelige og Mathematiske Afhandlingar. Deels ix. and x.	Ditto.
A System of Mineralogy, comprising the most recent Discoveries. By J. D. Dana.	The Author.
The Journal of Agriculture, and the Transactions of the Highland and Agricultural Society of Scotland, 1844, July and October.	The Society.
Scheikundige Onderzoekingen, gedaan in het Laboratorium der Utrechtsche Hoogeschool. Deel. ii., Stuk. 5.	The Editors.
Report of the Thirteenth Meeting of the British Association for the Advancement of Science, held at Cork in August 1843.	The Association.
The Eleventh Annual Report of the Royal Cornwall Polytechnic Society, 1843.	The Society.
The Journal of the Royal Geographical Society of London. Vol. xiv., Part 1.	Ditto.
Proceedings of the Royal Astronomical Society. Vol. vi., Nos. 1-6.	Ditto.
The Electrical Magazine, conducted by Mr Charles V. Walker. Vol. i., No. 5.	The Editor.
Journal of the Asiatic Society of Bengal. No. 143.	The Society.
Journal of the Bombay Branch Royal Asiatic Society. Nos. 5 and 6.	Ditto.
Annales des Sciences Physiques et Naturelles, d'Agriculture et d'Industrie, publiées par la Société Royale d'Agriculture, &c., de Lyon. Tome vi.	Ditto.
Geologische Bemerkungen über die Gegend von Baden bei Rastadt. Von J. F. L. Hausmann.	The Author.
Mémoire sur le Daltonisme. Par Elie Wartmann.	Ditto.
Astronomische Nachrichten, herausgegeben von H. C. Schumacher.	The Editor.
Versuch einer objectiven Begründung der Lehre von den drei Dimensionen des Raumes. Von Dr Bernard Bolzano.	The Author.
Magnetische und Meteorologische Beobachtungen zu Prag. Von Karl Kreil—(vierter Jahrgang).	Ditto.
Astronomical Observations made at the Royal Observatory, Greenwich, in the year 1842, under the direction of George Biddell Airy, Esq., M.A., Astronomer-Royal.	Royal Observatory.

DONATIONS.	DONORS.
Catalogue of the Places of 1439 Stars, referred to the 1st of January 1840; deduced from the Observations made at the Royal Observatory, Greenwich, from 1836, January 1, to 1841, December 31.	Royal Observatory.
Proceedings of the Geological Society of London. No. 98.	The Society.
Mémoires de la Société Géologique de France. (2 ^{me} Série). Tome i., 1 ^{re} Partie.	Ditto.
Sixth, Seventh, and Eighth Letters on Glaciers. By Professor Forbes.	The Author.
Proceedings of the Zoological Society of London. Nos. 120 to 134.	The Society.
Abhandlungen der Königl. Akademie der Wissenschaften zu Berlin aus dem Jahre 1842.	The Academy.
Bericht über die zur Bekanntmachung geeigneten Verhandlungen der Königl. Preuss. Akademie der Wissenschaften zu Berlin. Juli 1843 bis Juni 1844.	Ditto.
Tijdschrift voor Natuurlijke Geschiedenis en Physiologie—Uitgegeven door J. van der Hoeven & W. H. D. Vriese, M.D. Deel. xi., St. 2.	The Editors.
Archief voor Geneeskunde. Uitgegeven door Dr J. P. Heije. Deel. iii., St. 4.	The Editor.
Het Instituut of Verslagen en Mededeelingen, uitgegeven door de vier Klassen van het K. Nederlandsche Instituut van Wetenschappen, Letterkunde en Schoone Kunsten over den Jahre 1842, St. 4. 1843, St. 1, 2, 3.	Royal Institute of Holland.
Nieuwe Verhandelingen van het Bataafsch Genootschap, der Proefondervindelijke Wijsbegeerte te Rotterdam. Deel. ix., St. 1, 2, 3.	The Society.
Mémoire de l'Académie Impériale de Sciences de St. Pétersbourg—(Sciences Politiques, &c.) Tome vi., Liv. 4, 5, 6. Tome vii., Liv. 1, 2, 3.	The Imperial Academy.
— (Sciences Mathématiques.) Tome v., Liv. 4, 5, 6. Tome vi., Liv. 1.	Ditto.
Recueil des Actes de la Séance Publique de l'Académie Impériale de Sciences de St. Pétersbourg, tenue le 29. Dec. 1843.	Ditto.
Nouveaux Mémoires de la Société Helvétique des Sciences Naturelles. Tome i.—vi.	The Society.
Actes de la Société Helvétique des Sciences Naturelles.	Ditto.
Verhandlungen der Schweizerischen Naturforschenden Gesellschaft bei ihrer Versammlung zu Zurich, 1841.	Ditto.
— zu Altdorf, 1842.	Ditto.
Specimens of Printing-Types in the Establishment of Neill & Co., Printers, Edinburgh.	Messrs Neill & Co
Comptes Rendus Hebdomadaires des Séances de l'Académie des Sciences. Tome xviii., Nos. 15–26, and Tome xix., Nos. 1–16.	The Academy.
Maps of the Irish Ordnance Survey, containing the County of Limerick, 62 sheets.	The Lord Lieutenant.

December 16, 1844.

Annuaire de l'Académie Royale des Sciences et Belles Lettres de Bruxelles, 1844.	The Academy.
Bulletin de l'Académie Royale de Bruxelles. Tome x., Nos. 8–12. Tome xi., Nos. 1–8.	Ditto.
Mémoires Couronnés et Memoires des Savants Etrangers, publiées par l'Académie Royale des Sciences et Belles Lettres de Bruxelles. Tome xvi.	Ditto.
Annales de l'Observatoire Royale de Bruxelles. Par A. Quételet. Tome iii.	Ditto.
Annuaire de l'Observatoire Royale de Bruxelles. Par A. Quételet, 1844.	The Author.
Recherches Statistiques. Par A. Quételet.	Ditto.
Notices sur Pierre Simons, Alexis Bouvard, et Antoine Reinhard Falek. Par A. Quételet.	Ditto.
Bulletin de la Société Géologique de France. Tome i. Feuilles 28–33.	The Society.
Novi Commentarii Academiae Scientiarum Instituti Bononiensis. Vols. i., ii., iii., iv., v.	The Academy.
Opere Edite e Inedite del Professore Luigi Galvani, raccolte e pubblicate per cura dell' Accademia delle Scienze dell' Institute di Bologna.	The Academy.
On the Excision of the Eyeball in cases of Melanosis, Medullary Carcinoma, and Carcinoma, with Remarks by J. Argyll Robertson, M.D., F.R.S.E.	The Author.

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- The Journal of Agriculture, and the Transactions of the Highland Agricultural Society of Scotland, for January 1845. The Society.
- Arsberättelse om Zoologiens Framsteg under åren 1840-42. Af S. Loven. The Academy.
- Arsberättelse om Framstegen i Kemi och Mineralogi afgiven den 31 Mars 1844. Ditto.
- Af Jac. Berzelius.
- Arsberättelse om Botaniska Arbeten och Upptäckter för år. 1838. Af J. E. Wikström. The Academy.
- Kongl. Vetenskaps-Academiens Handlingar, för år. 1842. Ditto.
- Öfversigt af Kongl. Vetenskaps-Academiens Förhandlingar, 1844. Nos. 1 to 7. Ditto.
- The Journal of the Royal Asiatic Society. No. 15, Parts 1, 2. The Society.
- Observations Météorologiques faites à Nijne-Taguilsh (Monts Oural) Gouvernement de Perm. Année 1842. Sir Thomas M. Brisbane.
- Mittlere Oerter von 12,000 Fix Sternen, von Carl Rumker. Part 1, pp. 1-17. Ditto.

January 20.

- Journal of the Royal Asiatic Society of Bengal. Nos. 144, 145. The Society.
- On the Nature of the Nervous Agency. By James Stark, M.D., F.R.S.E. The Author.
- Researches on the Brain, Spinal Cord, and Ganglia, with Remarks on the Mode by which a continued flow of Nervous Agency is excited in, and transmitted from, these Organs. By James Stark, M.D., F.R.S.E. Ditto.
- Philosophical Transactions of the Royal Society of London for 1844. The Royal Society.
- Proceedings of the Royal Society of London. No. 59. Ditto.
- Magnetical and Meteorological Observations made at the Royal Observatory, Greenwich, in the year 1842, under the direction of George Biddell Airy, Esq., M.A., Astronomer-Royal. Ditto.
- Outlines of Chemistry, for the use of Students. By William Gregory, M.D. The Author.

February 3.

- The Electrical Magazine, conducted by Mr Charles V. Walker, for October 1844. The Editor.
- Memoir of Francis Baily, Esq., D.C.L., Oxford and Dublin. By Sir John F. W. Herschel, Bart. The Royal Astronomical Society.
- Inest de Stella Lyræ variabili Disquisitio. Per F. G. A. Argelander. Ditto.
- Description of Bones, &c., found near the River Ohio, 1786, with an Engraving, and Observations on the Annual passage of Herrings. By Mr John Gilpin. Anonymous.
- From the Columbian Magazine, December 1786.
- Three Volumes, in the Chinese Character, on Astronomy and Geography. Professor Forbes.

February 17.

- List of Specimens of Birds in the Collection of the British Museum. Parts 1 and 3. The Trustees of the British Museum.
- Aecipitres, Gallinæ, Grallæ, and Anseres. Ditto.
- List of the Specimens of Lepidopterous Insects in the Collection of the British Museum. Part 1. Ditto.
- List of the Specimens of Myriapoda in the Collection of the British Museum. Ditto.
- Catalogue of the Tortoises, Crocodiles, and Amphibæniæ, in the Collection of the British Museum. Ditto.
- The Electrical Magazine, conducted by Mr Charles V. Walker. Vol. i., No. 7. The Editor.
- Tijdschrift voor Natuurlijke Geschiedenis en Physiologie—Uitgegeven door J. van der Hoeven, M.D., & W. H. de Vriese, M.D., Deel. xi. St. 3, 4. The Editors.
- Cast of the Bust of the late Professor Playfair, which was executed by the late Sir Francis Chantrey. Sir George Mackenzie, Bart.
- Fifteenth Report of the Scarborough Philosophical Society. The Society.

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- The Journal of the Royal Agricultural Society of England. Vol. v., Part 2. The Society.
 Transactions of the Society for the Encouragement of Arts, Manufactures, and Ditto.
 Commerce. Vol. lv.
 Memoirs and Proceedings of the Chemical Society. Part ii. Ditto.
 The Journal of Agriculture, and the Transactions of the Highland and Agricultural Society of Scotland, for March 1845. The Society.
 Fifteenth Report of the Scarborough Philosophical Society, for the year 1844. Ditto.

March 17.

- Anatomical and Pathological Observations. By John Goodsir, F.R.S.E., and The Authors.
 Harry D. S. Goodsir, M.W.S.
 The American Journal of Science and Arts, conducted by Professor Silliman, for The Editor.
 January 1845.

April 7.

- Scheikundige Onderzoekingen, gedaan in het Laboratorium der Utrechtsche Hoogeschool. 2^{de} Deel. 6^{de} Stuk. The Editors.
 The Journal of the Royal Geographical Society of London. Vol. xiii. Part 2, The Society.
 and Vol. xiv., Part 2.
 The London University Calendar 1845. The University.
 Account of the Northumberland Equatorial Dome attached to the Cambridge Observatory. Duke of Northumberland.
 Observations made at the Magnetical and Meteorological Observatory at Toronto The British Government.
 in Canada, printed by order of Her Majesty's Government under the superintendence of Lieut.-Col. Edward Sabine, of the Royal Artillery.
 Memoir of Thomas Henderson, Esq., Professor of Practical Astronomy in the University of Edinburgh. By Thomas Galloway, Esq. The Author.
 The Grasses of Britain. Part 2. By Richard Parnell, M.D., F.R.S.E. Ditto.
 On the Chemical Constitution of the Bones of the Vertebrated Animals. By James Stark, M.D., F.R.S.E. Ditto.
 Memoirs and Proceedings of the Chemical Society. Part 12. The Society.
 The Fifth and Ninth Letters on Glaciers. By Professor Forbes, F.R.S.S.L. & E. The Author.
 On the Medicinal properties of Bebeerine. By Douglas MacLagan, M.D., F.R.S.E. Ditto.
 Remarks on the Improvements of Tidal Rivers. By David Stevenson, C.E. Ditto.
 On a possible explanation of the Adaptation of the Eye to distinct Vision at different distances. By Professor Forbes, F.R.S.S.L. & E. Ditto.

April 21.

- Journal of the Statistical Society of London. Vol. viii., Part 1. The Society.
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 Bulletin de la Société de Géographie. (2^{me} Série.) Tome xvi., xvii., xviii. The Society.
 Comptes Rendus Hebdomadaires des Séances de l'Académie des Sciences. Tome xix., The Academy.
 Nos. 17-27. Tome xx., Nos. 1-11.

December 1.

- Vestiges of the Natural History of Creation. The Author.
 Report of the Fourteenth Meeting of the British Association for the Advancement of Science, held at York, in September 1844. The Association.
 De l'Influence Curative du Climat de Pau et des Eaux Minérales des Pyrénées. The Author.
 Par M. A. Taylor, M.D.
 A Catalogue of the Library of the Athenæum. The Athenæum.

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The Transactions of the Royal Irish Academy. Vol. xx.	The Society.
Journal of the Statistical Society of London. Vol. viii., Parts 2, 3.	Ditto.
Memoirs and Proceedings of the Chemical Society. Parts 13, 15.	Ditto.
Outlines of Chemistry, for the Use of Students. Part 2. By William Gregory, M.D.	The Author.
Geschiedenis der Ioden in Nederland. Door M. H. J. Koenen.	
Over Het Onmatig Gebruik van Sterken Drank en de Middelen om Helzelve te Keer te Gaan. Door A. W. F. Herckenroth.	
Het Gebruik en Misbruik der Geestrijke Dranken. Door H. M. Duparc.	The Directors of the Provincial Society of Arts and Sciences at Utrecht.
De Uitoefening de Geregte Geneeskunde in Nederland. Door J. C. Van Den Broecke.	
Uitkomsten der Meteorologische Waarnemingen, gedaan te Utrecht, in de Jaren 1839-43.	
Natuurkundige Verhandelingen van de Hollandsche Maatschappij der Wetenschappen te Haarlem.	The Society.
The American Journal of Science and Arts, conducted by Professor Silliman and Benjamin Silliman jun., for April, July, and October.	The Editors.
A Physiological Essay on the Thymus Gland. By John Simon, Esq., F.R.S.	The Author.
On the Comparative Anatomy of the Thyroid Gland. By John Simon, Esq., F.R.S.	Ditto.
Astronomical Observations made at the Royal Observatory, Greenwich, in the year 1843, under the direction of George Biddell Airy, Esq., Astronomer-Royal.	The Royal Society.
Reduction of the Observation of Planets, made at the Royal Observatory, Greenwich, from 1750 to 1830; computed by order of the Lords Commissioners of the Treasury, under the superintendence of George Biddell Airy, Esq., Astronomer-Royal.	Ditto.
Philosophical Transactions of the Royal Society of London for the year 1845. Pt. 1.	The Royal Society.
Proceedings of the Royal Society, 1844. No. 60.	Ditto.
Transactions of the Geological Society of London. (2d Series.) Vol. vii., Parts 1, 2.	The Society.
Proceedings of the Geological Society of London. Nos. 99, 100, and 101.	Ditto.
Annuaire Magnétique et Météorologique du Corps des Ingénieurs des Mines de Russie. Par A. T. Kupffer. 1842. Nos. 1, 2.	The Author.
Etudes sur la Mortalité dans les Bagnes et dans les Maisons Centrales de Force et de Corrections de France depuis 1822 jusqu'à 1837. Par M. R. Chassinat, M.D.	Ditto.
Resultats des Observations Magnétiques faites à Genève dans les années 1842 et 1843. Par E. Plantamour, Professeur d'Astronomie à l'Académie de Genève.	Ditto.
Astronomische Nachrichten herausg. von H. C. Schumacher. Nos. 536, 537, 538.	Ditto.
The Electrical Magazine, conducted by Mr Charles V. Walker. Vol. ii., No. 9.	Ditto.
Journal of the Asiatic Society of Bengal, edited by the Secretary. Nos. 146 to 154.	The Society.
Carte Géologique du Globe. Par M. A. Boué.	The Geological Society of France.
The Twelfth Annual Report of the Royal Polytechnic Society, 1844.	The Society.
Address to the Ethnological Society of London, delivered at the Anniversary Meeting. By Richard King, M.D.	The Author.
Scheikundige Onderzoekingen, gedaan in het Laboratorium der Utrechtsche Hoogeschool. Deel. iii., St. 1, 2.	The Editors.
Tijdschrift voor Natuurlijke Geschiedenis en Physiologie. Uitgegeven door J. Van der Hoeven, M.D., & W. H. D. Vriese, M.D. Deel. xii., St. 1, 2.	The Editors.
On the Vision of Objects on and in the Eye. By William Mackenzie, M.D.	The Author.
Journal of the Bombay Branch Royal Asiatic Journal. April and October 1843.	The Society.
Bulletin de la Société de Géographie (2 ^{me} Série), Tomes xvi., xvii., xviii., xix., and xx.; and Tomes i., ii. (3 ^{me} Série.)	Ditto.
Mémoires de la Société de Physique et d'Histoire Naturelle de Genève. Tome x., ptie. 2.	Ditto.
Memoire della Reale Accademia delle Scienze di Torino. Vol. xxxix.	Ditto.
Handbuch der Mineralogie. Von J. F. L. Hausmann, Zweite Theil.	The Author.
Transactions of the Zoological Society of London. Vol. ii., Parts 2, 3, 4, 5; and Vol. iii., Parts 1, 2, 3.	The Society.
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Abhandlungen der Königl. Gesellschaft der Wissenschaften zu Göttingen. Band 2.	Ditto.
Nieuwe Verhandelingen der Eerste Klasse van het Koninkl. Nederlandsche Instituut van Wetenschappen, Letterkunde en Schoone Kunsten te Amsterdam. Deel. ii.	The Institute.
Annales des Sciences Physiques et Naturelles d'Agriculture et d'Industrie, Publiées par la Société Royale d'Agriculture, &c., de Lyon. Tome vii.	The Society.
Pilote Français; comprenant les Cotes Septentrionales de France depuis les Roches de Porsal jusqu' au Phare des Heaux de Brehat. 6 ^{me} Partie.	The French Government.
Pilote Français; Instructions Nautiques (Partie des Cotes Septentrionales de France comprise entre la Pointe de Barfleur et Dunkerque et entre les Casquets et la Pointe de Barfleur Environs de Cherbourg), Rédigées par M. Givry. 2 Parties, 4to.	Ditto.
Proceedings of the Geological Society of London. No. 103.	The Society.
Proceedings of the American Philosophical Society. Nos. 30 and 31.	Ditto.
A Public Discourse in commemoration of Peter S. Du Ponceau, LL.D., late President of the American Philosophical Society. By Robley Duglison, M.D.	Ditto.
Archæologia, or Miscellaneous Tracts relating to Antiquity, published by the Society of Antiquaries of London. Vols. i., ii., xi., xii., xiv., xvi., xvii., xviii., xix., xx., xxi., xxii., xxiii., xxiv., xxv., xxvi., xxvii., xxviii., xxix., xxx., & Index.	The Antiquarian Society of London.
Liber Quotidianus Contrarotularis Garderobæ anno Regni Regis Edwardi Primi, vigesimo octavo, A.D. 1299 et 1300.	Ditto.
A Catalogue of Ordinances and Regulations for the Government of the Royal Household, made in divers Reigns, from King Edward III. to King William and Queen Mary; also Receipts in Ancient Cookery.	Ditto.
Magni Rotuli Seaccarii Normanniæ sub Regibus Angliæ. Opera T. Stapleton. 2 vols. 8vo.	Ditto.
Cædmon's Metrical Paraphrase of Parts of the Holy Scriptures, in Anglo-Saxon; with an English Translation. By Benjamin Thorpe.	Ditto.
Urologie. Des Angusties ou Rétrécissements de l'Urètre et de leur traitement rationnel. Par le Dr Leroy-D'Etoilles.	The Author.
Sullo Studio Comparativo della Lingue Osservazioni Generali di B. Biondelli.	Ditto.
Etudes Philologiques et Historiques. Par M. Halbertsma. 2 Parties.	Ditto.
Instructions Pratiques sur l'Observations et la mesure des Propriétés optiques appelées Rotatoires, avec l'exposé succinct de leur application à la Chimie Médicale, Scientifique et Industrielle. Par M. Biot.	Ditto.
Maps of the Ordnance Survey of the County of Cork.	The Lord-Lieutenant of Ireland.
Comptes Rendus Hebdomadaires des Séances de l'Académie des Sciences. Tome xx., Nos. 12-26; and Tome xxi., Nos. 1-16.	The Academy.
Proceedings of the Philosophical Society of Glasgow. Nos. 1-11.	The Society.
December 15, 1845.	
Journal of the Asiatic Society of Bengal, 1844. No. 145.	Ditto.
The Transactions of the Linnean Society of London. Vol. xix., Parts 3, 4.	Ditto.
Proceedings of the Linnean Society of London. Nos. 19 to 26.	Ditto.
Archives du Muséum d'Histoire Naturelle, publiées par les Professeurs-Administrateurs de cet Etablissement. (Paris.) Tome iv., Livres 1, 2.	The Editors.
The 2d Annual Report of the Agricultural and Horticultural Society of Auckland. Waarnemingen en Proeven over de onlangs Geheerscht Hebbende Ziekte der Aardappelen. Door G. Vrolik, M.D.	The Society.
Œuvres de La Place, 4 Tomes.	The French Government.

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- Recueil des Actes de la Séance Publique de l'Académie Impériale des Sciences de St Pétersbourg, tenue le 29 Decembre 1844. The Academy.
- Mémoires de l'Académie Impériale des Sciences de St Pétersbourg (Sciences Politique, &c.) Tome v., Livres 5 and 6. The Academy.
- Mémoires de l'Académie Imp. des Sciences de St Pétersbourg (Sciences Mathématiques, &c.) Tome iv., Livres 6. Ditto.
- Mémoires de l'Académie Impériale des Sciences de St Pétersbourg (présentés par divers Savans.) Tome iv., Livres 6. Ditto.
- Catalogue of the Edinburgh Subscription Library, 1794-1846. The Directors.
- Journal of the Statistical Society of London. Vol. viii., Part. 4. The Society.
- Proceedings of the Geological Society of London. Vol. iv., Nos. 104. Ditto.
- The Journal of Agriculture, and the Transactions of the Highland and Agricultural Society of Scotland. January 1846. Ditto.
- Journal of the Asiatic Society of Bengal, Nos. 146 and 1847. Ditto.
- The Derivation of many Classical Proper Names from the Gaelic Language, or the Celtic of Scotland. By Thomas Stratton, M.D. The Author.

January 19.

- Third Bulletin of the Proceedings of the National Institute for the Promotion of Science at Washington. February 1842 to February 1845. The Institute.
- Tenth Letter on Glaciers. By Prof. Forbes. The Author.
- Notes on the Topography and Geology of the Cuchullin Hills in Skye, and on the Traces of Ancient Glaciers which they present. By Professor Forbes. Ditto.
- Journal of the Asiatic Society of Bengal. No. 158. The Society.
- The Electrical Magazine. Conducted by Mr Charles V. Walker. October 1845. The Editor.
- The Journal of the Royal Geographical Society of London. Vol. xv., Part 2. The Society.
- Bulletin de la Société de Géographie. Tome iii., 3^{me} Série. Ditto.
- Abhandlungen der Königlichen Akademie der Wissenschaften zu Berlin. 1843. Ditto.
- Bericht über die zur Bekanntmachung geeigneten Verhandlungen der Königl. Preuss. Akademie der Wissenschaften zu Berlin. Juli 1844 bis June 1845. The Academy.
- Novorum Actorum Academiæ Cesariæ Leopoldino-Carolinæ Naturæ Curiosorum, Vol. 19 Supplementum et Vol. 20. Ditto.
- Scheikundige Onderzoekingen gedaan in het Laboratorium der Utrechtsche Hoogeschool. Deel. iii., St. 3. The Editors.

February 2.

- Annuaire de l'Observatoire Royal de Bruxelles. Pour 1845. Par A. Quételet. The Observatory.
- Annuaire de l'Académie Royale des Sciences et Belles Lettres de Bruxelles, pour 1845. The Academy.
- Bulletins des Séances de l'Académie Royale des Sciences et Belles Lettres de Bruxelles, 1844. Nos. 9, 10, 11, 12, and 1845, Nos. 1, 2, 3, 4, 5, 6. Ditto.
- Annales des l'Observatoire Royal de Bruxelles. Tome iv. Par A. Quételet. Ditto.
- Nouveaux Mémoires de l'Académie R. des Sciences et Belles Lettres de Bruxelles. Tome xvii. et xviii. Ditto.
- Mémoires Couronnées et Mémoires des Savants Etrangers publiées par l'Académie Royale des Sciences et Belles Lettres de Bruxelles. Tome xvii. et xviii. Ditto.
- Mémoire de Simon Stevin. Par A. Quételet. The Author.
- The Geology of Russia in Europe and the Ural Mountains. By Sir Roderick Impey Murchison, Edouard de Verneuil, and Count Alexander von Keyserling. 2 vols. 4to. The Authors.
- The American Journal of Arts and Science, for January 1846. Conducted by Professor Silliman. The Editor.

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Journal of the Asiatic Society of Bengal. No. 161.	The Society.
Comptes Rendus Hebdomadaires des Séances de l'Académie des Sciences. Tome xxi., Nos. 17-25, et Tome xxii., Nos. 1, 2.	The Academy.
The Electrical Magazine, conducted by Mr Charles V. Walker. January 1846.	The Editor.
Proceedings of the Royal Society of London. No. 61.—Philosophical Transactions of the Royal Society of London, 1845. Part 2.	The Royal Society.
Catalogue of Stars of the British Association for the Advancement of Science; containing the mean Right Ascensions and North Polar Distances of Eight Thousand three hundred and seventy-seven, reduced to January 1, 1850, with a Preface explanatory of their construction and application. By the late Francis Baily, D.C.L., President of the Royal Astronomical Society of London.	The British Association.
Magnetical and Meteorological Observations made at the Royal Observatory, Greenwich, in the year 1843; under the direction of George Biddell Airy, Esq., M.A., Astronomer-Royal.	The Royal Society.
Konigl. Vetenskaps Akademiens Handlingar för Ar. 1843.	The Academy.
Arsberättelse om Zoologiens Framsteg under Aren 1840-42. Första Delen af C. J. Sundeval.	Ditto.
Arsberättelse om Zoologiens Framsteg under Aren 1843-44. Andra Delen af C. H. Bohemen.	Ditto.
Arsberättelse om Botaniska Arbeten och Upptäckter af J. E. Wikström.	Ditto.
Arsberättelse om Framstegen i Kemi och Mineralogi af Jac. Berzelius.	Ditto.
Leçons de Géologie Pratique. Par L. Elie de Beaumont. Tome i.	The Author.
<i>March 2.</i>	
The Journal of Agriculture, and the Transactions of the Highland and Agricultural Society of Scotland, for March 1846.	The Society.
Journal of the Asiatic Society of Bengal. No. 160, for 1845.	Ditto.
Life and Correspondence of David Hume. From the Papers bequeathed by his Nephew to the Royal Society of Edinburgh, and other original sources. By John Hill Burton, Esq., Advocate. 2 vols. 8vo.	The Author.
Natural History of New York. 10 vols. 4to. Geological Map of New York, published by Legislative authority in 1842.	The Government of New York.
<i>March 16.</i>	
The London University Calendar, 1846.	The University.
Journal of the Asiatic Society of Bengal. No. 159.	The Society.
The Electrical Magazine. Conducted by Mr Charles V. Walker. January 1846.	The Editor.
Twenty-fifth Report of the Council of the Leeds Philosophical and Literary Society for Session 1844-45.	The Society.
Biographical Notice of the late Sir John Robison, K.H., Sec. R.S.E. By Prof. Forbes.	The Author.
Il Cimento; Giornale di Fisica, Chimica e Storia Naturale. 1844 & 1845, Jan. to Aug.	Prof. Forbes.
Nieuwe Verhandelingen der Eerste Klasse van het K. Nederlandsche Instituut van Wetenschappen, Letterkunde en Schoone Kunsten te Amsterdam. Deel. xii., St. 1.	The Institute.
<i>April 6.</i>	
Proceedings of the American Philosophical Society. Vol. iv., Nos. 32 and 33.	The Society.
Transactions of the American Philosophical Society held at Philadelphia, for promoting Useful Knowledge. (New Series), Vol. ix., Part 2.	Ditto.
Flora Batava. Nos. 139 and 140.	The King of the Netherlands.
Journal of the Statistical Society of London. Vol. ix., Part 1. March 1846.	The Society.
The American Journal of Arts and Science, for March 1846, conducted by Professor Silliman, B. Silliman junior, and James D. Dana.	The Editors.

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The Quarterly Journal of the Geological Society. Vol. i., and Vol. ii., Part 1.	The Society.
Proceedings of the Royal Astronomical Society. Vol. vi., Nos. 9 to 17, and Vol. vii., Nos. 1, 2, 3.	Ditto.
Memoirs of the Royal Astronomical Society. Vol. xv.	Ditto.
Meteorological Observations for 1842 and 1843, made at the Bombay Government Observatory. By George Buist, LL.D.	The Author.
Magnetic Observations made at the Bombay Government Observatory from May 1842 to December 1843. By George Buist, LL.D.	Ditto.
Tracings of the Wind-Gauge for 1842 and 1843, made at the Bombay Government Observatory from May 1842 to Dec. 1843. By George Buist, LL.D.	Ditto.
Journal of the Asiatic Society of Bengal. No. 162.	The Society.
Memoirs and Proceedings of the Chemical Society. Part 16.	Ditto.
Maps of the Geological Survey of the United Kingdom of Great Britain. By Sir H. T. De la Beche, Director-General of the Geological Survey.	The Author.

April 20, 1846.

Metaphysical Analysis, revealing, in the Process of the Formation of Thought, a new Doctrine of Metaphysics. By J. W. Tombs.	The Author.
Novorum Actuum Academiæ Cæsareæ Leopoldino-Carolinæ Naturæ Curiosorum. Vol. xxi., Pars 1.	The Academy.
Comptes Rendus Hebdomadaires des Séances de l'Académie des Sciences. Tome xxii., Nos. 2—12.	Ditto.
Journal of the Asiatic Society of Bengal. No. 163.	The Society.
A work on the Science of Mathematics, embracing Conic Sections, Perspective, &c. By Nuwab Shums-ool-oomiah of Hyderabad; the Illustrations lithographed by the Author.	Dr Burt.

December 7.

The Electrical Magazine, conducted by Mr Charles V. Walker, for April, July, and October 1846.	The Editor.
Annuaire de l'Observatoire Royale de Bruxelles, pour l'Année 1846, par le Directeur A. Quételet.	The Author.
Lettres au Duc Régnant de Saxe-Cobourg et Gotha, sur la Théorie des Probabilités, appliquée aux Sciences Morales et Politiques, par A. Quételet.	Ditto.
Annuaire de l'Académie Royale des Sciences des Lettres et des Beaux Arts de Belgique pour 1846.	The Academy.
Bulletins de l'Académie R. des Sciences et Belle Lettres de Bruxelles. Tome xii. Pte 2.	Ditto.
Proceedings of the Royal Society. Nos. 62, 63, 64, and 65.	The Royal Society.
Philosophical Transactions of the Royal Society of London. 1846. Parts 1, 2, 3.	Ditto.
The Thirteenth Annual Report of the Royal Polytechnic Society.	The Society.
Structure and Classification of Zoophytes. By James D. Dana, A.M.	The Author.
United States Exploring Expedition—Zoophytes. By James D. Dana.	The Author.
Nachrichten von der Georg-Augusts Universitäts und der Königl. Gesellschaft der Wissenschaften zu Göttingen. Von Juli bis December 1845.	The Society.
Handbuch der Mineralogie von J. F. L. Hausmann, 2 Theil. 3 Abtheilung.	The Author.
Flora Batava. Nos. 141, 142, and 143.	The King of the Netherlands.
Report of the Fifteenth Meeting of the British Association for the Advancement of Science, held at Cambridge in 1845.	The British Association.
Journal of the Asiatic Society of Bengal. Nos. 164, 167.	The Society.
Journal of the Statistical Society of London. Vol. ix., Parts 2, 3.	Ditto.
Journal of the Royal Asiatic Society. Vol. x., Part 1.	Ditto.
Bulletin de la Société de Géographie. (3 ^{me} Série), Tome iv. et v.	Ditto.
Archæologia; or, Miscellaneous Tracts relating to Antiquity; published by the Society of Antiquaries of London. Vol. xxxi.	Ditto.

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Memoirs of the American Academy of Arts and Science. (New Series), Vol. ii.	The Academy.
Bulletin des Séances de la Société Vaudoise des Sciences Naturelles. Tome i.	The Society.
Monthly Prize Essays. Vol. i., No. 1.	The Director.
The Journal of Agriculture, and the Transactions of the Highland and Agricultural Society of Scotland, for July and October.	The Society.
Proceedings of the Zoological Society of London, January 14 to December 9, 1845.	Ditto.
Memoirs of the Geological Survey of Great Britain, and of the Museum of Economic Geology in London. Vol. i.	The Lords Commissioners of Her Majesty's Treasury.
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